RELATIVITY CHALLENGE

RelativityChallenge.com

Episode 17
Einstein's 1905 Special Relativity Theory Derivation

Presented by Steven Bryant

Objectives

- Increase your understanding of Section 3 of Einstein's 1905 derivation
- Walk through Section 3 of Einstein's 1905 derivation
  - Show how the equations should have been presented and derived (modernized version)
  - Contrast against how Einstein originally derived the equations
- Summarize key problems with Einstein's 1905 derivation

- Note: The focus is on clarifying the mathematics, not on modifying the descriptive supporting text

Foundational RelativityChallenge.com Podcast Episodes

- **Episode 8** – Tau is a Function
- **Episode 16** – Moving Systems Foundational Equations
- **Episode 15** – Moving Systems (Part 4) – A look at Special Relativity
- **Episode 9** – The Importance of Distinguishing between Lengths and Points
- **Storrs Conference Presentation**

Einstein's 1905 Derivation

If we place $x' = x - vt$, it is clear that a point at rest in the system k must have a system of values x', y, z, independent of time. We first define τ as a function of x', y, z, and t. To do this we have to express in equations that τ is nothing else than the summary of the data of clocks at rest in system k, which have been synchronized according to the rule given in § 1.

From the origin of system k let a ray be emitted at the time τ_0 along the X-axis to x', and at the time τ_1 be reflected thence to the origin of the co-ordinates, arriving there at the time τ_2; we then must have $\frac{1}{2}(\tau_0 + \tau_2) = \tau_1$, or, by inserting the arguments of the function τ and applying the principle of the constancy of the velocity of light in the stationary system:—

$$\frac{1}{2}\left[\tau(0,0,0,t) + \tau\left(0,0,0,t + \frac{x'}{c-v} + \frac{x'}{c+v}\right)\right] = \tau\left(x',0,0,t + \frac{x'}{c-v}\right).$$

Key Points

- Einstein uses the Newtonian equation $x' = x - vt$ as part of his derivation
- Einstein properly uses time "instance variables"; $\tau_0, \tau_1, and\ \tau_2$
- τ is a function, not an equation, and is better written as $\tau(x, y, z, t)$ or as $\tau()$

Modernized 1905 Derivation

Since τ is a *linear* function, it follows from these equations that

$$\tau(x', y, z, t) = \left\{ \alpha(t - \frac{vx'}{c^2 - v^2}) \right\},$$

which can be written equivalently as

$$\tau(k, l, m, n) = \left\{ \alpha(n - \frac{vk}{c^2 - v^2}) \right\},$$

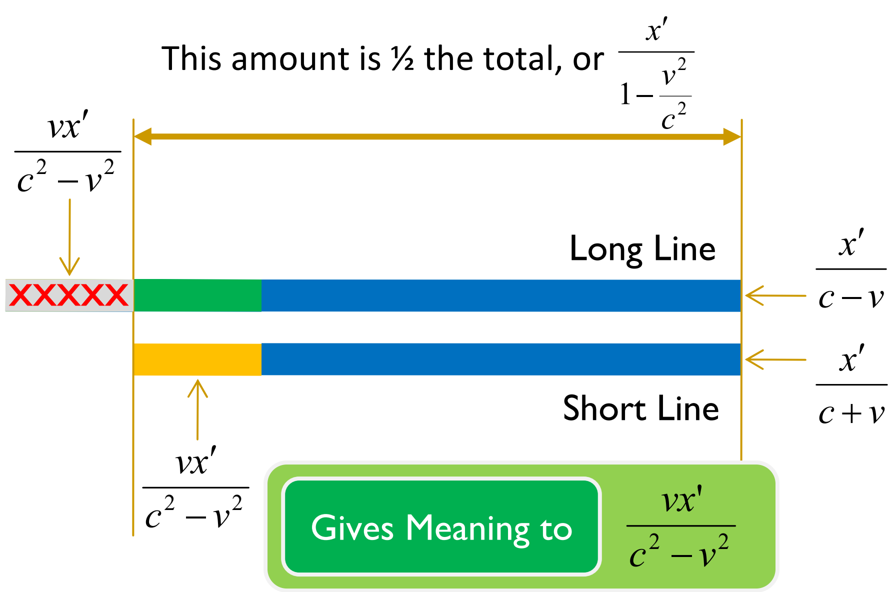
where α is a function $\phi(v)$ at present unknown, and where for brevity it is assumed that at the origin of k, $\tau() = 0$, when $\tau(0,,,0)$.

Key Points

- The $\tau()$ function is written in formal terms
- You must understand all aspects of this function to truly understand Einstein's derivation. This function returns the length of time required to complete ½ of one oscillation based on given parameters
- The first parameter is the amount of length covered by the ray of light in one direction along the X axis with respect to the moving system
- The fourth parameter is the amount of time that the ray of light takes to cover the distance (along the X axis) specified by the first parameter, with this time measured with respect to the reference system
- The second and third parameters are not used

Answering “How far is half way”?

There are three ways to mathematically answer the question: *“How long does it take to travel ½ the total round-trip distance?”*



1. Subtract the short line from the long line
2. Divide remainder into two equal parts
3. Either subtract from long line or add to short line

- Three ways to find ½ the total round trip time:
  - Add $\frac{x'}{c+v}$ to $\frac{x'}{c-v}$ and divide by 2
  - Subtract $\frac{vx'}{c^2-v^2}$ from $\frac{x'}{c-v}$
  - Add $\frac{vx'}{c^2-v^2}$ to $\frac{x'}{c+v}$
- When you multiply a “bi-directional” time by velocity, you get a length

Key Finding

Einstein answers the questions **“How far is ½ the total round trip distance”?** when he states:

$$\xi = c\left[\frac{x'}{c-v}-\frac{vx'}{c^2-v^2}\right]=\frac{x'c^2}{c^2-v^2}$$

Modernized 1905 Derivation – Cont.

With the help of this result we easily determine the quantities ξ, η, ζ by expressing in equations that light (as required by the principle of the constancy of the velocity of light, in combination with the principle of relativity) is also propagated with velocity c when measured in the moving system. For a ray of light emitted at the time $\tau() = 0$ in the direction of increasing ξ

$$\xi = c\tau_\xi = c\tau(x', y, z, \frac{x'}{c-v}) = c\alpha(\frac{x'}{c-v} - \frac{vx'}{c^2-v^2}) = \alpha\frac{c^2x'}{c^2-v^2} = \alpha\frac{x'}{1-\frac{v^2}{c^2}},$$

noting that the ray moves relative to the initial point of k, when measured in the stationary system, with the velocity $c-v$, allowing the use of $\frac{x'}{c-v}$ as the fourth parameter.

Key Points

- The $\tau()$ function is formally invoked and properly uses τ_ξ as an instance variable
- Invocation creates an instance equation, facilitating the subsequent substitution of x' with $x-vt$ in a later step

Modernized 1905 Derivation – Cont.

In an analogous manner we find, by considering rays moving along the two other axes, that

$$\eta = c\tau_{\eta} = c\tau(0, y, z, \frac{y}{\sqrt{c^2 - v^2}}) = c\alpha(\frac{y}{\sqrt{c^2 - v^2}} - \frac{v*0}{c^2 - v^2}) = c\alpha\frac{y}{\sqrt{c^2 - v^2}} = \alpha\frac{y}{\sqrt{1 - \frac{v^2}{c^2}}}$$

when the fourth parameter is $\frac{y}{\sqrt{c^2 - v^2}}$ and the first parameter is 0 and

$$\zeta = c\tau_{\zeta} = c\tau(0, y, z, \frac{z}{\sqrt{c^2 - v^2}}) = c\alpha(\frac{z}{\sqrt{c^2 - v^2}} - \frac{v*0}{c^2 - v^2}) = c\alpha\frac{z}{\sqrt{c^2 - v^2}} = \alpha\frac{z}{\sqrt{1 - \frac{v^2}{c^2}}}$$

when the fourth parameter is $\frac{z}{\sqrt{c^2 - v^2}}$ and the first parameter is 0.

Key Points

- The $\tau()$ function is formally invoked and properly uses τ_{η} and τ_{ζ} as instance variables

Modernized 1905 Derivation – Cont.

Since we have already stated that $x' = x - vt$, substituting for x' its value and α with $\phi(v)$, we obtain

$$\xi = \phi(v)\beta^2(x - vt)$$
$$\eta = \phi(v)\beta y$$
$$\zeta = \phi(v)\beta z$$
$$\tau_\xi = \phi(v)\beta^2 \frac{(x - vt)}{c}$$
$$\tau_\eta = \phi(v)\beta \frac{y}{c}$$
$$\tau_\zeta = \phi(v)\beta \frac{z}{c}$$

where

$$\beta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

and ϕ is an as yet unknown function of v. If no assumption whatever be made as to the initial position of the moving system and to the zero point of $\tau()$, an additive constant is to be placed on the right side of each of these equations.

Key Points

- Correctly uses β^2 and β in each instantiated equation
- Equations conform with the rules of Algebraic substitution

Foundational Equations

Einstein's Moving Systems model is based on a core set of foundational equations that answer the question: *what is ½ the distance*?

Foundational Length Equations

$$x\ axis = \frac{x'}{1-\frac{v^2}{c^2}}$$

$$y\ axis = \frac{y}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$z\ axis = \frac{z}{\sqrt{1-\frac{v^2}{c^2}}}$$

Foundational Time Equations

$$x\ axis = \frac{x'}{c\left[1-\frac{v^2}{c^2}\right]}$$

$$y\ axis = \frac{y}{c\sqrt{1-\frac{v^2}{c^2}}}$$

$$z\ axis = \frac{z}{c\sqrt{1-\frac{v^2}{c^2}}}$$

Modernized 1905 Derivation – Cont.

For reference, Einstein replaces the local variable x' *with its value,* $x - vt$*, incorrectly simplifying the un-invoked* $\tau()$ *function as the equation*

$$\tau = \phi(v)\,\beta^2\,(t - \frac{vx}{c^2}).$$

While this equation could be derived as $\tau = \tau(x', y, z, t) = \alpha(t - \frac{vx'}{c^2 - v^2}) = \alpha(t - \frac{v(x - vt)}{c^2 - v^2}) = \alpha(\frac{t - \frac{vx}{c^2}}{1 - \frac{v^2}{c^2}})$,

such a function invocation is not supported by the mathematics or textual explanation in Einstein's derivation.

Key Points

- Correctly uses β^2

Original 1905 Derivation

Since τ is a *linear* function, it follows from these equations that

$$\tau = a\left(t - \frac{v}{c^2 - v^2}x'\right)$$

where a is a function $\phi(v)$ at present unknown, and where for brevity it is assumed that at the origin of k, $\tau = 0$, when $t = 0$.

Key Points

- The function is written in informal terms
- Einstein does not fully explain the operation of this function
- Einstein does not explain the meaning of the local variables *x'* and *t*
- Einstein's informal function specification makes it easy to mistake this function as an equation
- The function only returns 0 if *x'* and *t* are zero. Einstein only says *t* is zero.

Compare to Slide 5

Original 1905 Derivation – Cont.

With the help of this result we easily determine the quantities ξ, η, ζ by expressing in equations that light (as required by the principle of the constancy of the velocity of light, in combination with the principle of relativity) is also propagated with velocity c when measured in the moving system. For a ray of light emitted at the time $\tau = 0$ in the direction of the increasing ξ

$$\xi = c\tau \text{ or } \xi = ac\left(t - \frac{v}{c^2 - v^2}x'\right).$$

But the ray moves relatively to the initial point of k, when measured in the stationary system, with the velocity $c - v$, so that

$$\frac{x'}{c - v} = t.$$

If we insert this value of t in the equation for ξ, we obtain

$$\xi = a\frac{c^2}{c^2 - v^2}x'.$$

Key Points

- Einstein treats x' as a length. This suggests that $x - vt$ is also a length
- Einstein uses an informal invocation to properly derive ξ
- To avoid confusion τ should be specified as $\tau()$ to represent a function or as τ_ξ to represent an instance variable
- Einstein does not say that he assigns x'=x' as part of the informal invocation. Doing so, in my opinion, would have been confusing

Compare to Slide 7

Original 1905 Derivation – Cont.

In an analogous manner we find, by considering rays moving along the two other axes, that

$$\eta = c\tau = ac\left(t - \frac{v}{c^2 - v^2}x'\right)$$

when

$$\frac{y}{\sqrt{c^2 - v^2}} = t,\ x' = 0.$$

Thus

$$\eta = a\frac{c}{\sqrt{c^2 - v^2}}y \text{ and } \zeta = a\frac{c}{\sqrt{c^2 - v^2}}z.$$

Key Points

- Einstein uses an informal invocation to properly derive η and ζ
- It is easy to mistake τ as an algebraic variable. It should be defined as the instance variables τ_η and τ_ζ to avoid confusion

Compare to Slide 8

Original 1905 Derivation – Cont.

Substituting for x' its value, we obtain

$$\begin{aligned} \tau &= \phi(v)\beta(t - vx/c^2), \\ \xi &= \phi(v)\beta(x - vt), \\ \eta &= \phi(v)y, \\ \zeta &= \phi(v)z, \end{aligned}$$

where

$$\beta = \frac{1}{\sqrt{1 - v^2/c^2}},$$

and ϕ is an as yet unknown function of v. If no assumption whatever be made as to the initial position of the moving system and as to the zero point of τ, an additive constant is to be placed on the right side of each of these equations.

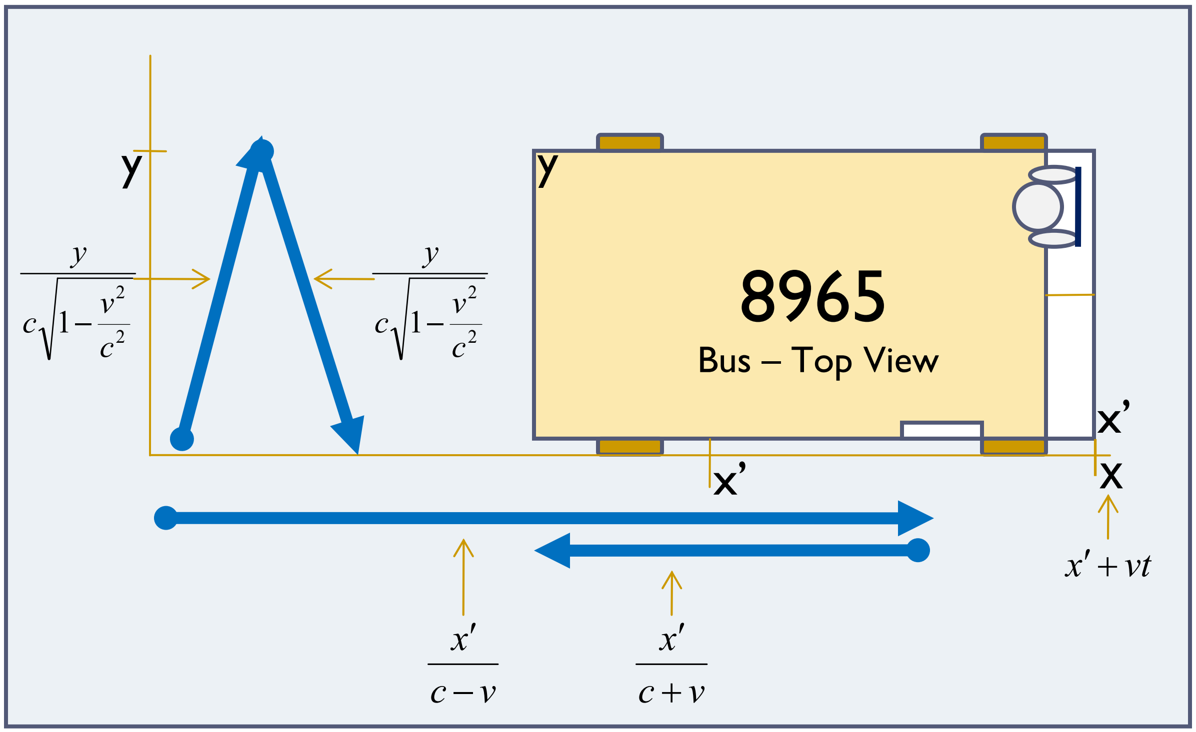
Key Points

- Einstein drops a β from the right-hand-side of each equation
- Einstein incorrectly simplifies τ as an equation rather than as a function
- Einstein does not use the 3 time equations representing length of time required to travel the given length along each axis

Compare to Slide 9

Our key conceptual diagram

Review Episodes of the RelativityChallenge.com Podcast series to understand the genesis of this diagram.



Note: The same equations that apply to the Y axis also apply to the Z axis.

Original 1905 Derivation – Cont.

Einstein's Math & Explanation	Formal Function Invocation
For a ray of light emitted at the time $\tau = 0$ in the direction of the increasing ξ $\xi = c\tau$ or $\xi = ac\left(t - \frac{v}{c^2 - v^2}x'\right)$.	$\tau(x', y, z, \frac{x'}{c-v})$
In an analogous manner we find, by considering rays moving along the two other axes, that $\eta = c\tau = ac\left(t - \frac{v}{c^2 - v^2}x'\right)$ when $\frac{y}{\sqrt{c^2 - v^2}} = t,\ x' = 0.$	$\tau(0, y, z, \frac{y}{\sqrt{c^2 - v^2}})$ $\tau(0, y, z, \frac{z}{\sqrt{c^2 - v^2}})$

Key Points

- The set (ξ, η, ζ) represents three lengths, not a single point
- Review Episode 16 to observe where, with respect to the moving system, the phenomena is when it reaches the time specified in the function invocation

Problems with Einstein's Original 1905 Derivation

- Incorrectly simplifies the time function
  - Algebraically substitutes $x - vt$ for x' and simplifies
  - Does not invoke the τ function before simplifying

- Incorrectly removes a β term from the final equations

- Does not explain how the τ function works or the meaning of each of the function parameters
- A problem in Einstein's derivation is found using Algebraic means
- Transforms lengths, not points

Thank You

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