RelativityChallenge.Com Podcast
Episode 7

The equations of Complete and Incomplete Coordinate Systems

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Coordinate System Definition

- A basic **Coordinate System** is something that can be measured along one, two, or three dimensions

- Characteristics
  1. One coordinate system can be put into motion with respect to another coordinate system
  2. An object can be put into motion to travel between points in a coordinate system
  3. There is some sort of medium on which, or through which, an object travels.

Coordinate Systems - Diagram

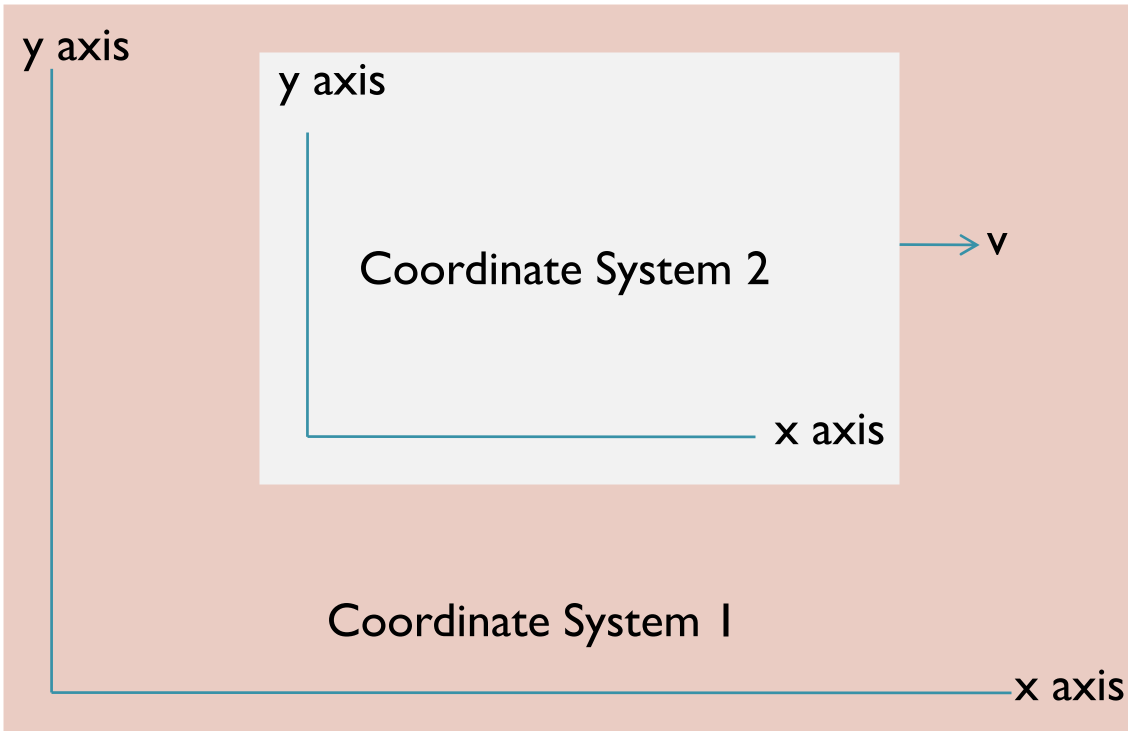


Figure 1

In this example, Coordinate System 1 will be the stationary, or reference system, and Coordinate System 2 will be the moving coordinate system (at velocity v)

Complete Coordinate System

- In a **Complete Coordinate System**, the object that is oscillating between points in the moving system travels *on, or through, a medium that is also in motion with regard to the moving system*

Incomplete Coordinate System

- In an **Incomplete Coordinate System**, the object that is oscillating between points in the moving system travels *on, or through, a medium that is in motion with regard to the stationary, or reference system*

Coordinate System - Example

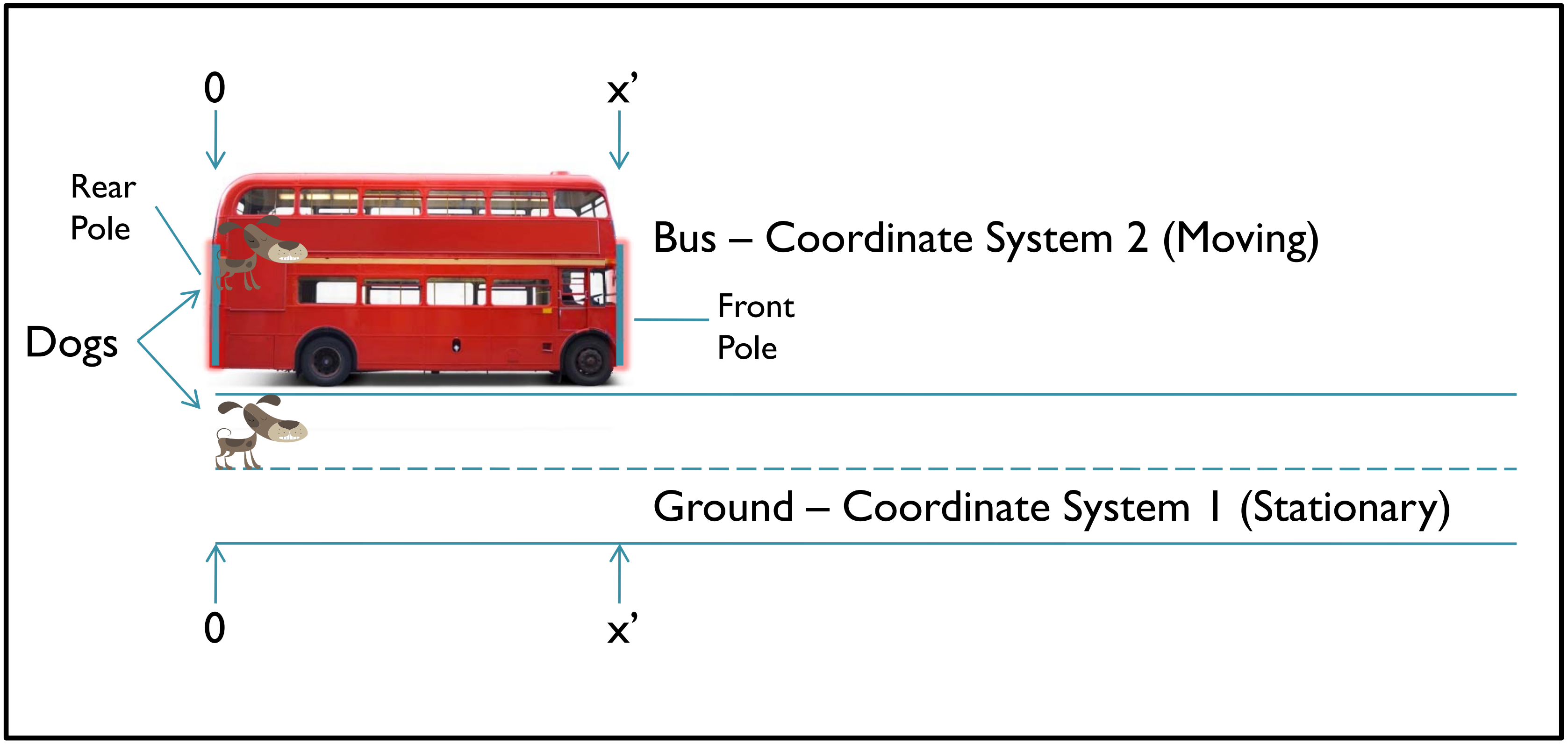
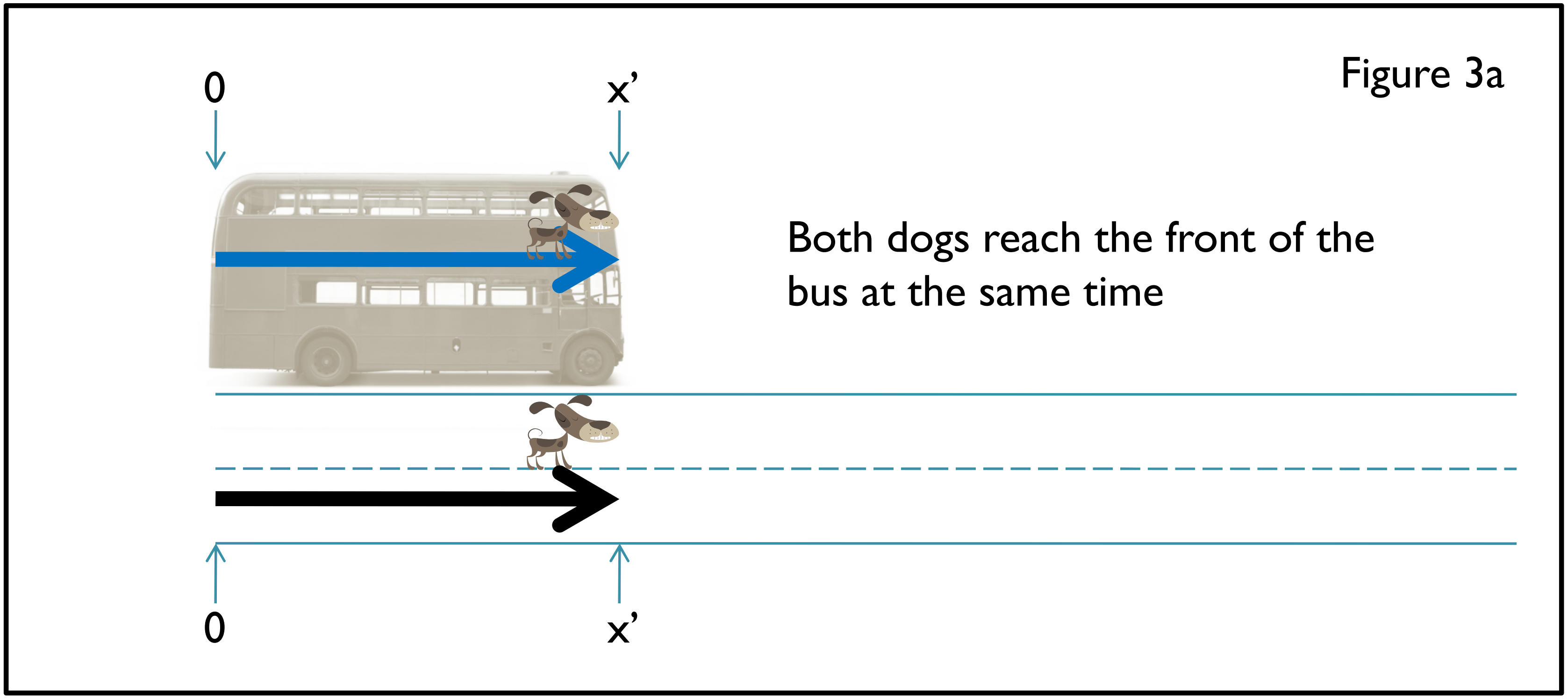


Figure 2

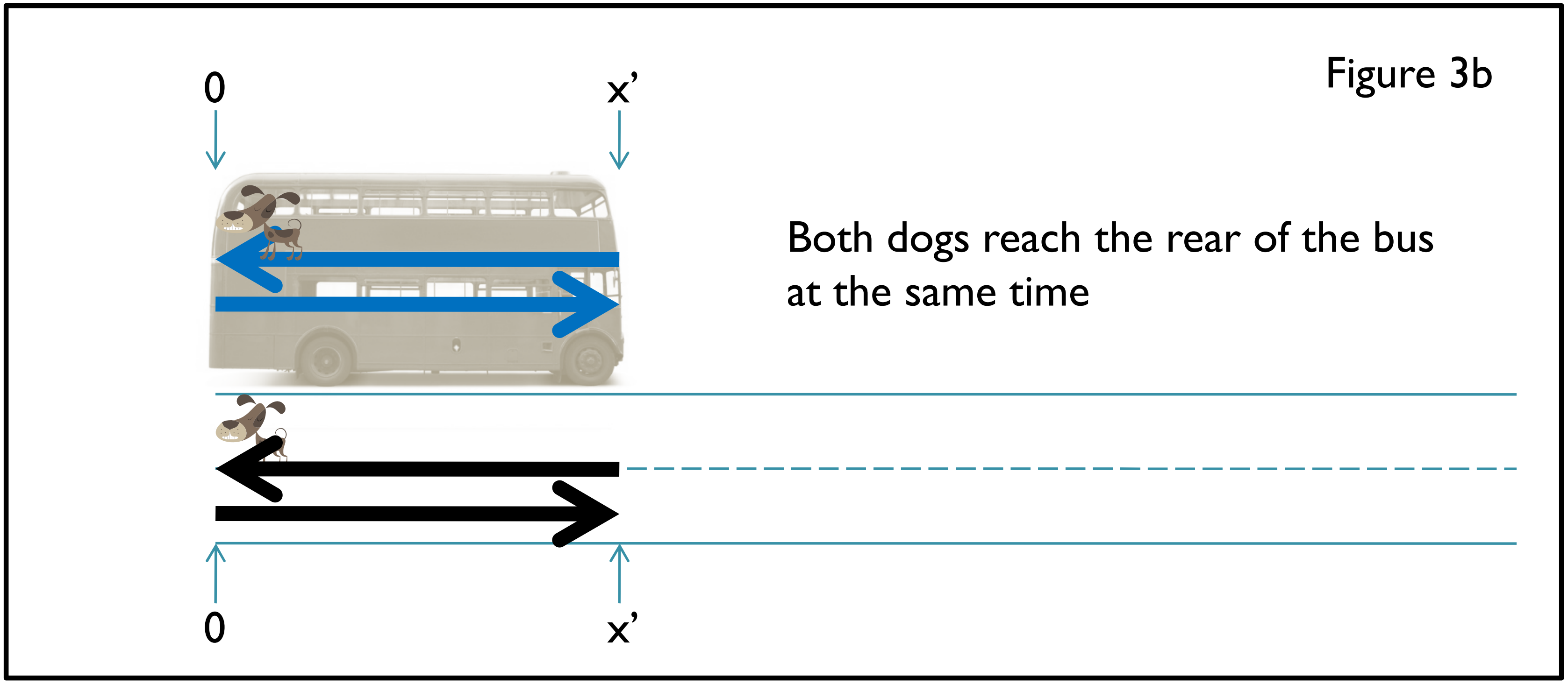
Note: Bus Driver & Pedestrian, while not illustrated, are useful in understanding which system we are measuring against

Baseline Measurements



- Bus is not moving
- Both dogs move at velocity w
- Distance from rear to front of bus is x’
- It takes time x’/w to travel to the front of the bus

Baseline Measurements

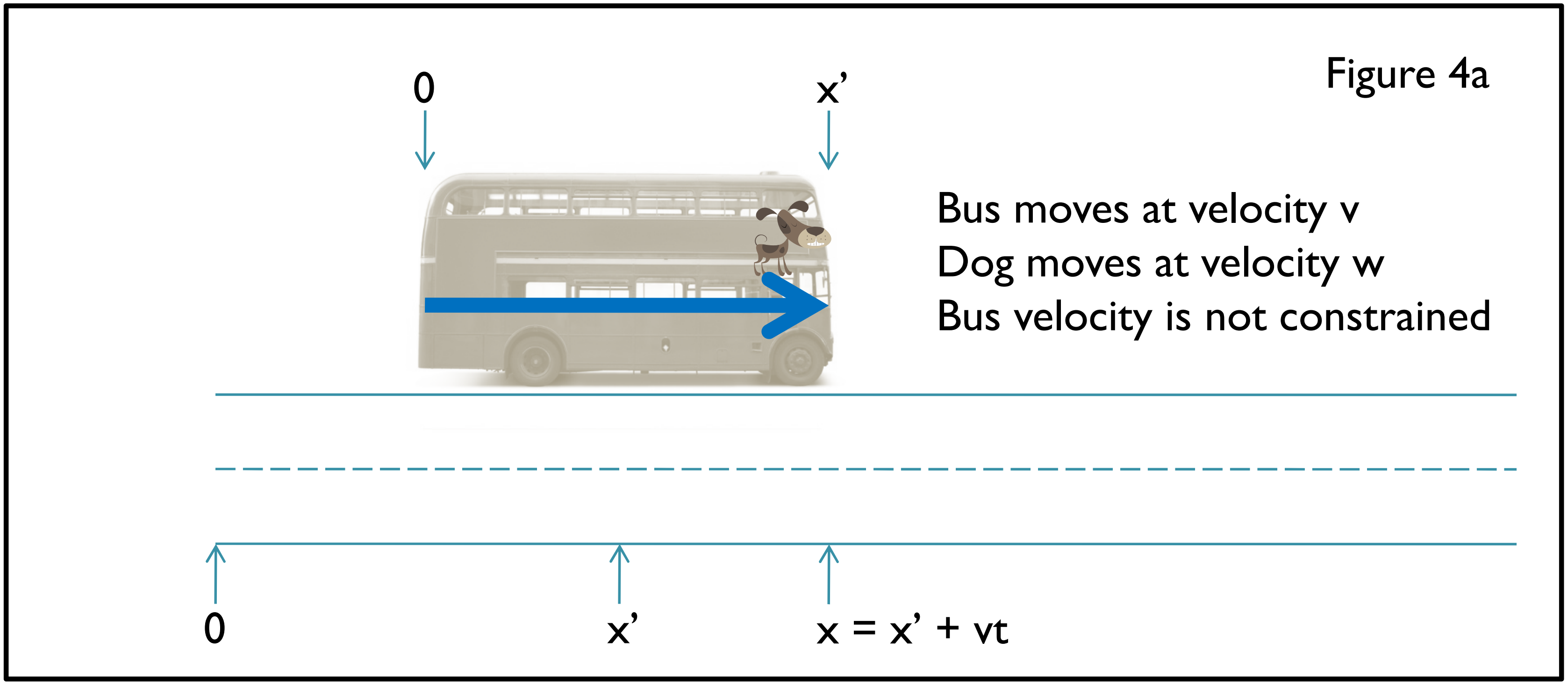


- Bus is not moving
- Both dogs move at velocity w
- Distance from front to rear of bus is x’
- It takes time x’/w to travel to the rear of the bus
- The total time for one “oscillation” is 2x’/w
- The total distance for one “oscillation” is 2wx’/w or simply 2x’

Baseline Measurements - Observations

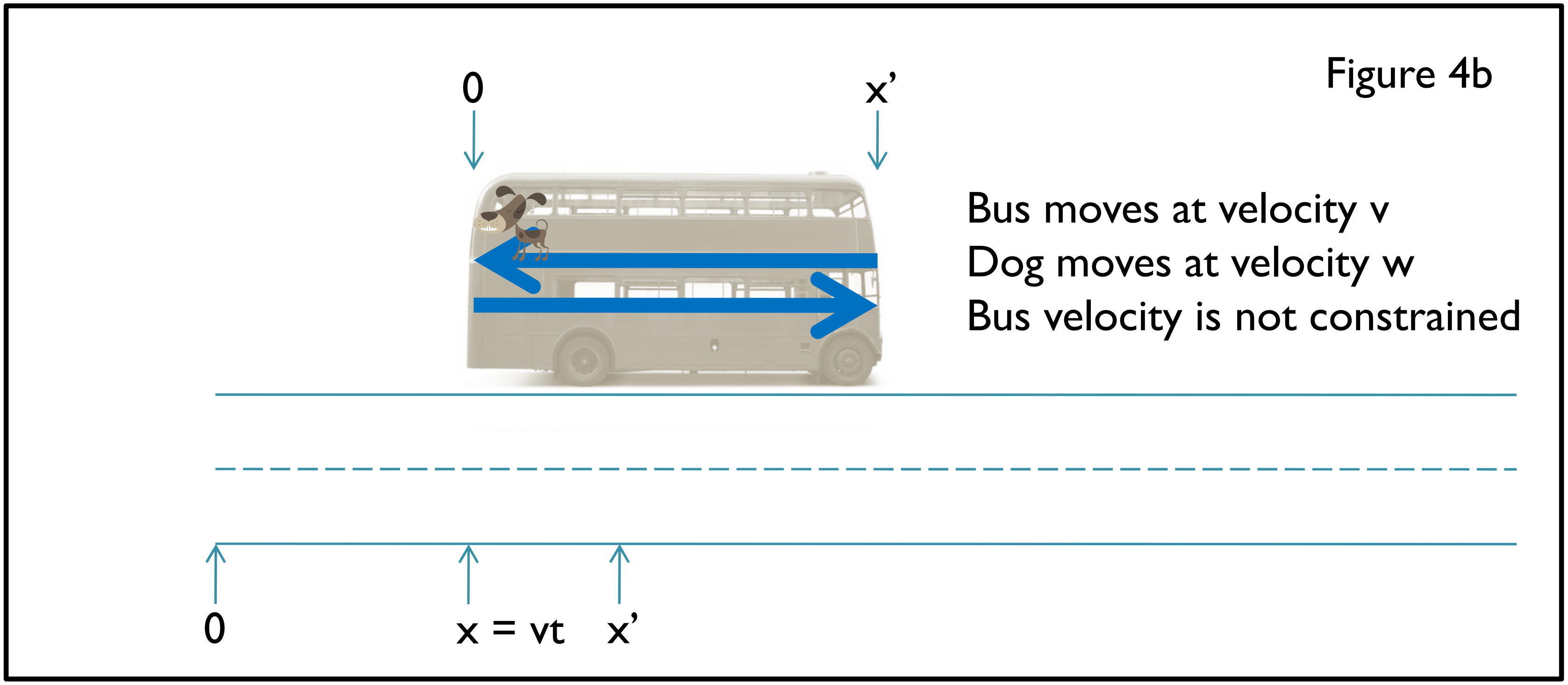
- The total distance run for one "oscillation" is 2x'
- One half of the total distance is x'
- When the objects (e.g., dogs) have traveled a distance of x', they have reached the front of the bus
- While the dog will travel a total distance of 2x' per trip, its position in either coordinate system after traveling this distance is not 2x' from the origin because the dog has traveled in two directions
- Distance is the amount of time to make the journey multiplied by the velocity of the object (e.g., dog)
- Time is the distance of the journey divided by the velocity of the object (e.g., dog)

Complete Coordinate System - Measurements



- Time to travel between the rear to the front of the bus is x'/w; and similarly for travel from the front to the rear
- When the dog reaches the front of the bus, the front of the bus is located at point x, where x = x' + v(x'/w), with respect to the ground.
- The dog has traveled a distance of x' as determined by the medium that it is traveling on

Complete Coordinate System - Measurements



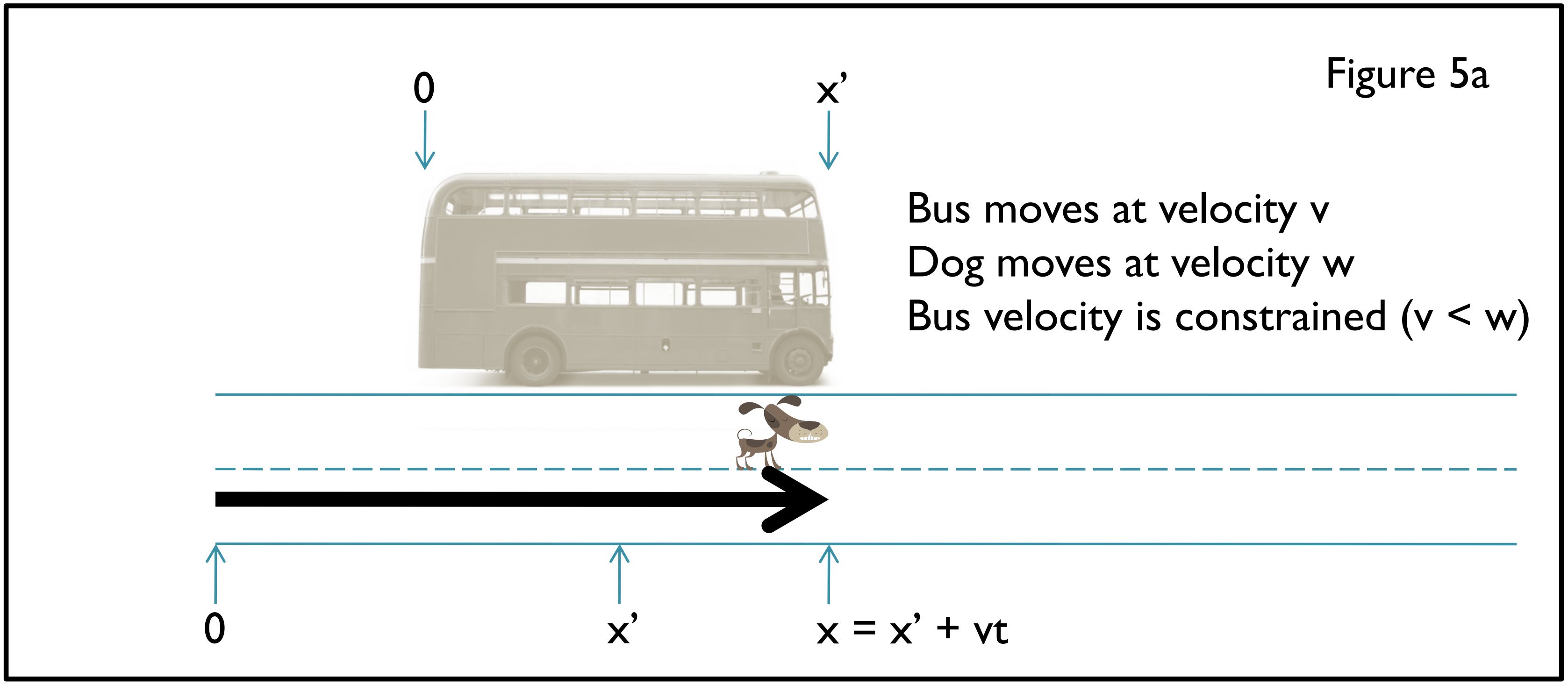
- When the dog reaches the rear of the bus, the rear is located at point x, where x = 0 + v(2x'/w), with respect to the ground
- The dog has traveled a distance of 2x' as determined by the medium that the dog is traveling on

- Note: In Fig. 4b, the position x is the result of an equation and can appear either to the left or to the right of x'

Complete Coordinate System - Observations

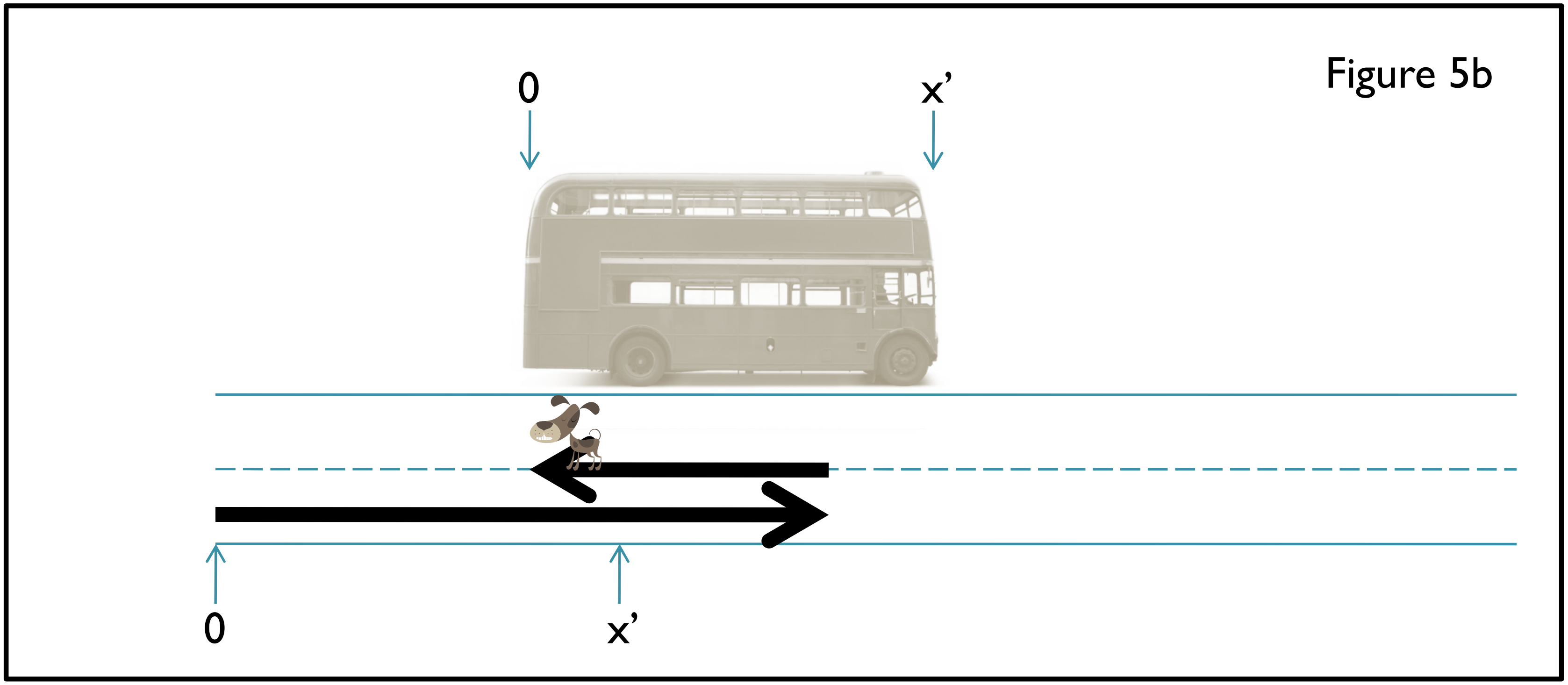
- Bus is moving at velocity v
- The energy / effort (as measured by distance or time) of the object is determine by *measurements in the moving system*
- The position of the front of bus, with respect to the reference system is found as x = x' + vt
- The position of the rear of the bus, with respect to the reference system is found as x = vt
- The object (e.g., dog) takes time x'/w to travel in either direction
- The object (e.g., dog) travels a distance of x' in either direction
- The total time for one "oscillation" is 2x'/w
- The total distance for one "oscillation" is 2wx'/w or 2x'

Incomplete Coordinate System - Measurements



- Time to travel between the rear to the front of the bus is x'/(w-v)
- When the dog reaches the front of the bus, the front of the bus is located at point x, where x = x' + v(x'/(w-v)).
- To reach the front of the bus, the dog has traveled a distance of wx'/(w-v) as determined by the medium that it is traveling on (e.g., the street)
- The *bus velocity must be less than w*, or the dog will not make it to the front

Incomplete Coordinate System - Measurements

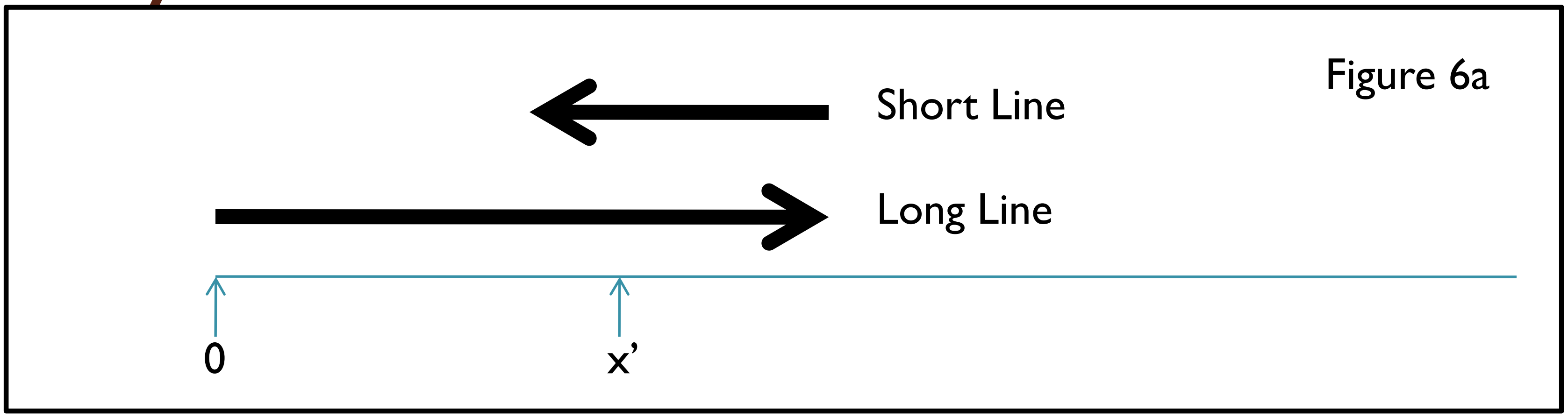


- Time to travel between the front to the rear of the bus is x'/(w+v)
- When the dog reaches the rear of the bus, the front bus is located at point x' + v(x'/(w-v)+x'/(w+v)) and the rear is located at v(x'/(w-v)+x'/(w+v))
- The dog has traveled a distance of wx'/(w+v) from the front of the bus to the rear of the bus, as determined by the medium that it is traveling on (e.g., the street)

Incomplete Coordinate System - Observations

- Bus is moving at velocity v
- The energy / effort (as measured by distance or time) of the object is determine by *measurements in the stationary system*
- The object (e.g., dog) takes longer travel from the rear to the front than from the front to the rear
- The object (e.g., dog) travels a longer distance from the rear to the front, than from the front to the rear
- The moving coordinate system (e.g., bus) must travel slower than the object (e.g., dog) or the dog will not be able to oscillate

Equations – Incomplete Coordinate Systems



Line	Description	Time	Length
Long Line	The amount of time, or distance, for the oscillating object traveling on, or through, a medium associated with the reference system	$\frac{x'}{w-v}$	$w\left[\frac{x'}{w-v}\right]$
Short Line	The amount of time, or distance, for the oscillating object traveling on, or through, a medium associated with the reference system	$\frac{x'}{w+v}$	$w\left[\frac{x'}{w+v}\right]$

Approach 1 – Finding the equations for ½ an oscillation

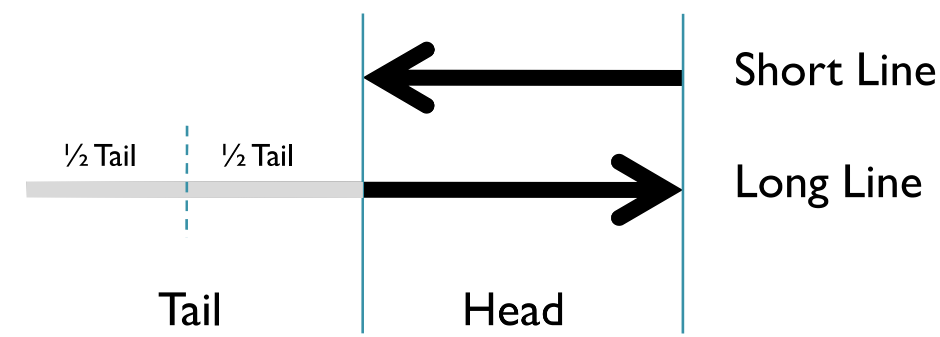
Description	Summary	Equation
Total Time – One Oscillation	Short Line Time + Long Line Time	$\frac{x'}{w+v}+\frac{x'}{w-v}$
Total Distance – One Oscillation	Short Line Distance + Long Line Distance	$w\left[\frac{x'}{w+v}+\frac{x'}{w-v}\right]$
½ Time of One Oscillation	[Short Line Time + Long Line Time] / 2	$\tau=\left[\frac{x'}{w+v}+\frac{x'}{w-v}\right]/2$
½ Distance of One Oscillation	[Short Line Distance + Long Line Distance] / 2	$\xi=w\left[\frac{x'}{w+v}+\frac{x'}{w-v}\right]/2$

Figure 6b

τ — Tau is ½ the time of one oscillation

ξ — Xi is ½ the distance of one oscillation

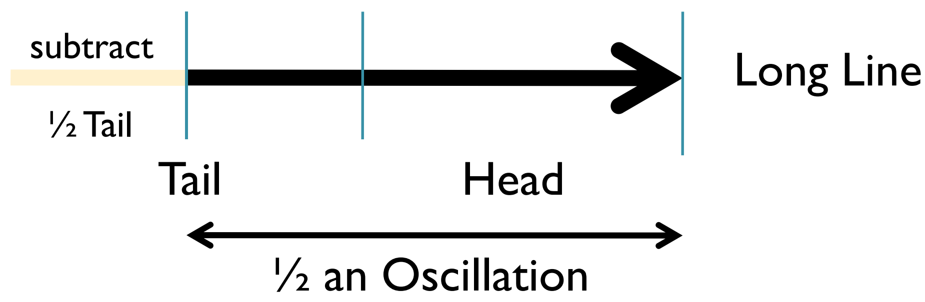
Equations – Incomplete Coordinate Systems



	Summary	Time
Tail (Time)	Long Line Time – Short Line Time	$\frac{2vx'}{w^2 - v^2}$
Tail (Distance)	Long Line Distance – Short Line Distance	$w\left[\frac{2vx'}{w^2 - v^2}\right]$
½ Tail (Time)	½ of the Tail (time)	$\frac{vx'}{w^2 - v^2}$
½ Tail (Distance)	½ of the Tail (distance)	$w\left[\frac{vx'}{w^2 - v^2}\right]$

Approach II
Finding the equations for ½ an oscillation

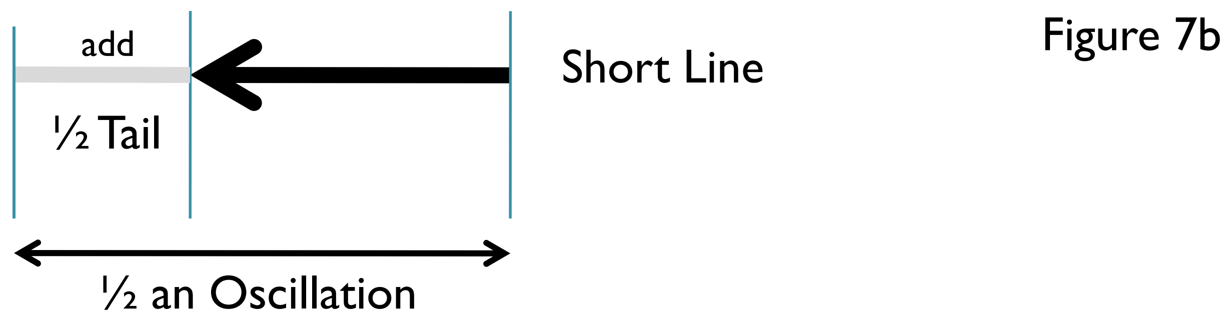
Figure 7c



Description	Summary	Equation
½ Time of One Oscillation	[Long Line Time - ½ Tail (Time)	$\left[\frac{x'}{w-v}-\frac{vx'}{w^2-v^2}\right]$
½ Distance of One Oscillation	[Long Line Distance - ½ Tail (Distance)	$w\left[\frac{x'}{w-v}-\frac{vx'}{w^2-v^2}\right]$

- Einstein uses this approach to find Xi, which is simply the distance of ½ an oscillation of an object in an Incomplete Coordinate System
- This is the Tau (Time) equation used in the model of Complete and Incomplete Coordinate Systems

Approach III
Finding the equations for ½ an oscillation



Description	Summary	Equation
½ Time of One Oscillation	[Short Line Time + ½ Tail (Time)	$\left[\frac{x'}{w+v}+\frac{vx'}{w^2-v^2}\right]$
½ Distance of One Oscillation	[Short Line Distance + ½ Tail (Distance)	$w\left[\frac{x'}{w+v}+\frac{vx'}{w^2-v^2}\right]$

Equation Summary

Variable	Meaning	Simplified	Equivalent Equations
ξ	½ of the distance of one oscillation of an object in an Incomplete Coordinate System	$\left[\dfrac{x'}{1-\dfrac{v^2}{w^2}}\right]$	$= w\left[\dfrac{x'}{w+v}+\dfrac{x'}{w-v}\right]/2$ $= w\left[\dfrac{x'}{w-v}-\dfrac{vx'}{w^2-v^2}\right]$ $= w\left[\dfrac{x'}{w+v}+\dfrac{vx'}{w^2-v^2}\right]$
τ	The amount of time required for the object to travel the distance represented by ξ	$\dfrac{1}{w}\left[\dfrac{x'}{1-\dfrac{v^2}{w^2}}\right]$	$=\left[\dfrac{x'}{w+v}+\dfrac{x'}{w-v}\right]/2$ $=\left[\dfrac{x'}{w-v}-\dfrac{vx'}{w^2-v^2}\right]$ $=\left[\dfrac{x'}{w+v}+\dfrac{vx'}{w^2-v^2}\right]$

Summary

At the end of today's presentation, I hope that you are now able to do the following…

1. Explain the difference between Complete and Incomplete Coordinate Systems
2. Understand how to find the equations for ½ and oscillation in a Complete and Incomplete Coordinate System
3. Explain the meaning of $\frac{vx'}{w^2 - v^2}$ and understand how it can be added to $\frac{x'}{w+v}$, or subtracted from $\frac{x'}{w+v}$ to find the equations for ½ an oscillation (time) or when the time equation is multiplied by w, for ½ an oscillation (distance)
4. If x' is not known, but x and t are known instead, then x' is found by using x'=x-vt.