RelativityChallenge.Com Podcast
Episode 8

Tau is a Function
Understanding the proper derivation of Einstein's 1905 Xi and Tau equations

Originally recorded: October 14, 2007

Steven Bryant

email@RelativityChallenge.com
www.RelativityChallenge.com
blog.RelativityChallenge.com

Beginning Comments

- You have to understand Functions in order to fully understand Einstein's derivation and the underlying problem.
- While Mathematics defines what a function does, it doesn't explore the anatomy of functions as rigorously as Computer Science.
  - Computer Science, and its core training and techniques, did not exist in 1905
  - Understanding the anatomy of functions is essential in understanding Einstein's 1905 derivation as well as identifying the problem
- Functions are not the same thing as equations
  - In mathematics, functions are often treated as if they were equations (often without a problem occurring)
  - In Einstein's derivation, the Tau function must be treated as function and not as equation to avoid a mathematical problem
- I will show Tau in a simplified form with two parameters instead of four, (since only two are used by the function). This will aid in the clarity of the presented material without affecting the validity of the discussion

What is a function?

- Mathematics Definition: A function maps elements of one domain (or set) into another domain (or set)

- While this definition explains "what" a function is, it is incomplete
  - Must address the "structure" of a function
  - Must address the proper "use" of a function
  - Functions provide an additional layer of "abstraction" not found in equations

Tau is a function

In Einstein's 1905 paper, he properly defines Tau as a function

Aus diesen Gleichungen folgt, da τ eine *lineare* Funktion ist:

$$\tau = a\left(t - \frac{v}{V^2 - v^2} x'\right),$$

Original text

Since τ is a *linear* function, it follows from these equations that

$$\tau = a\left(t - \frac{v}{c^2 - v^2} x'\right)$$

Translated text

Functions and equations

What distinguishes a function from an equation?

$$\tau = \alpha(t - \frac{vx'}{c^2 - v^2})$$ The Linear Function

Versus

$$\tau = \alpha(t - \frac{vx'}{c^2 - v^2})$$ The Equation

Formal and Informal Functions

Functions, especially when written informally, can be mistreated as an equation

Informal function specification	$\tau = \alpha(t - \frac{vx'}{c^2 - v^2})$
Semi-formal function specification	$\tau() = \alpha(t - \frac{vx'}{c^2 - v^2})$
Formal function specification	$\tau(x', t) = \alpha(t - \frac{vx'}{c^2 - v^2})$

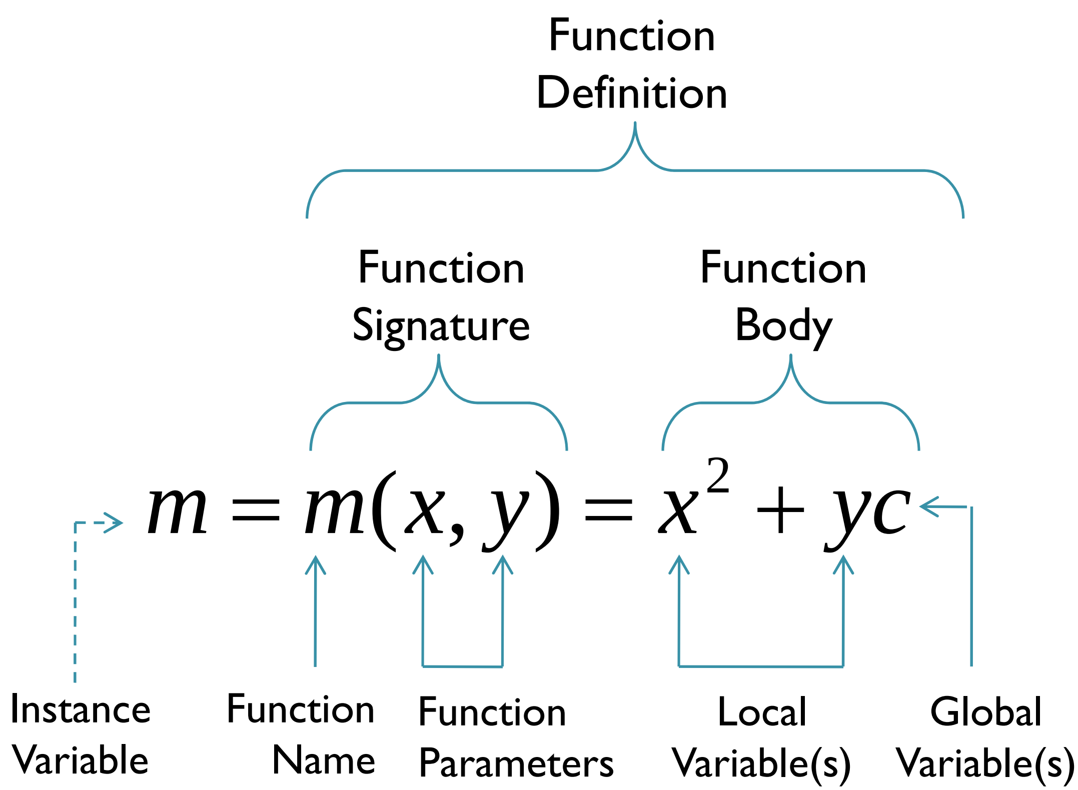
Informal: We have not additional information indicating that it is a function

Semi-formal: We know it is a function, but know nothing about the variables

Formal: We know it is a function and we know about the variables

Anatomy of a function

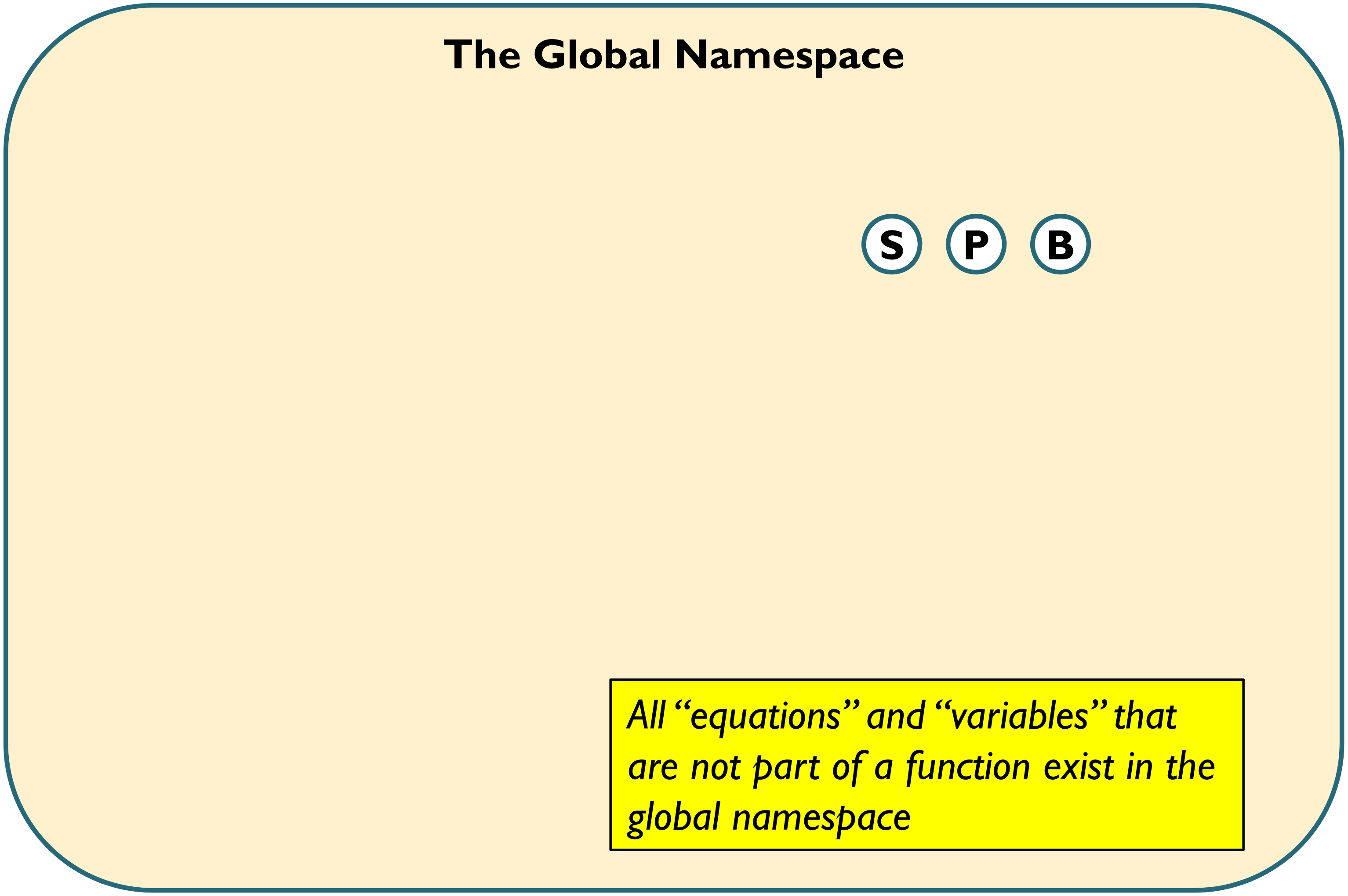
A function consists of many separate parts, making them different than their equation counterpart



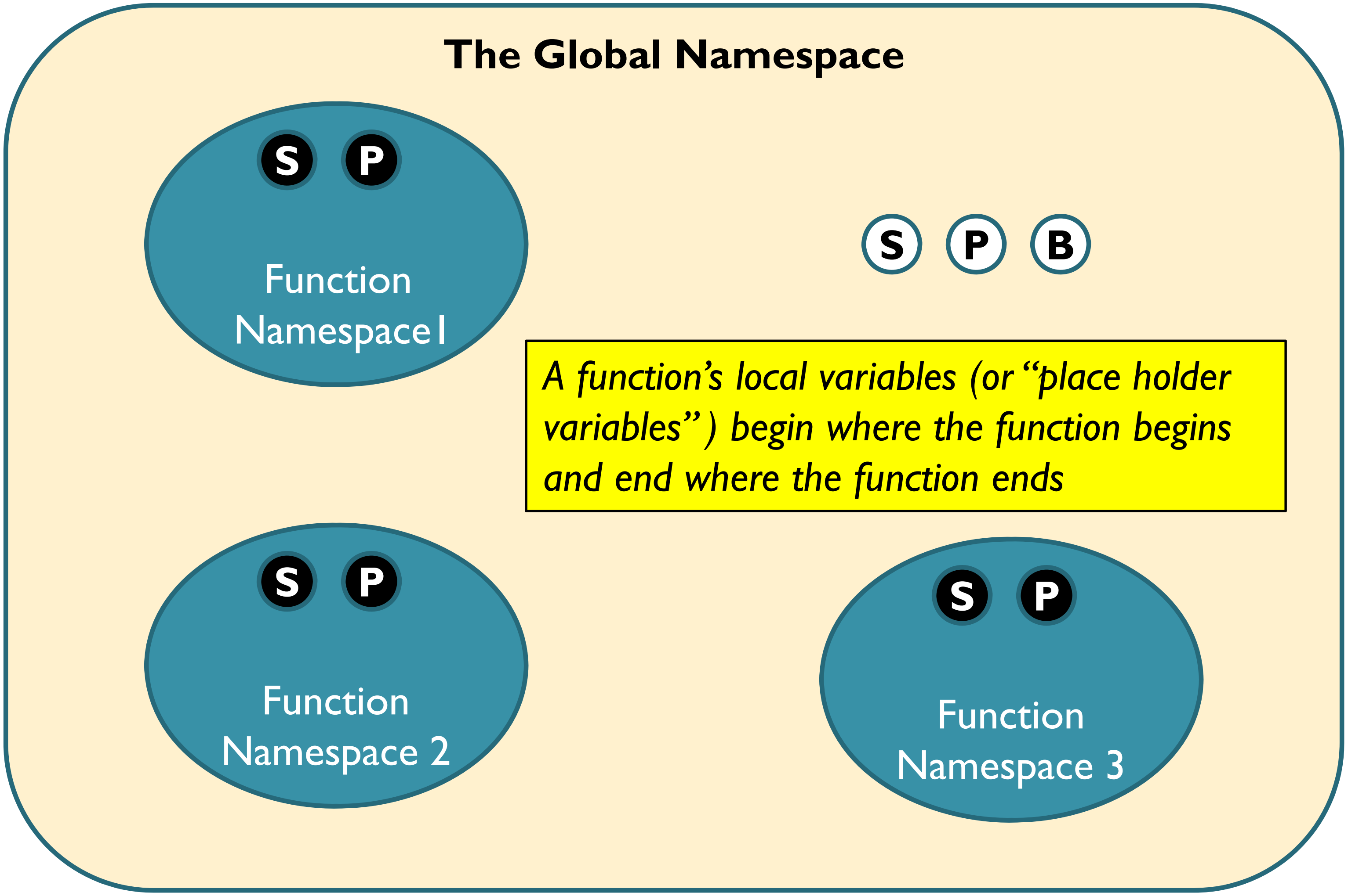
Functions and function variables Key Concepts

The Global Namespace

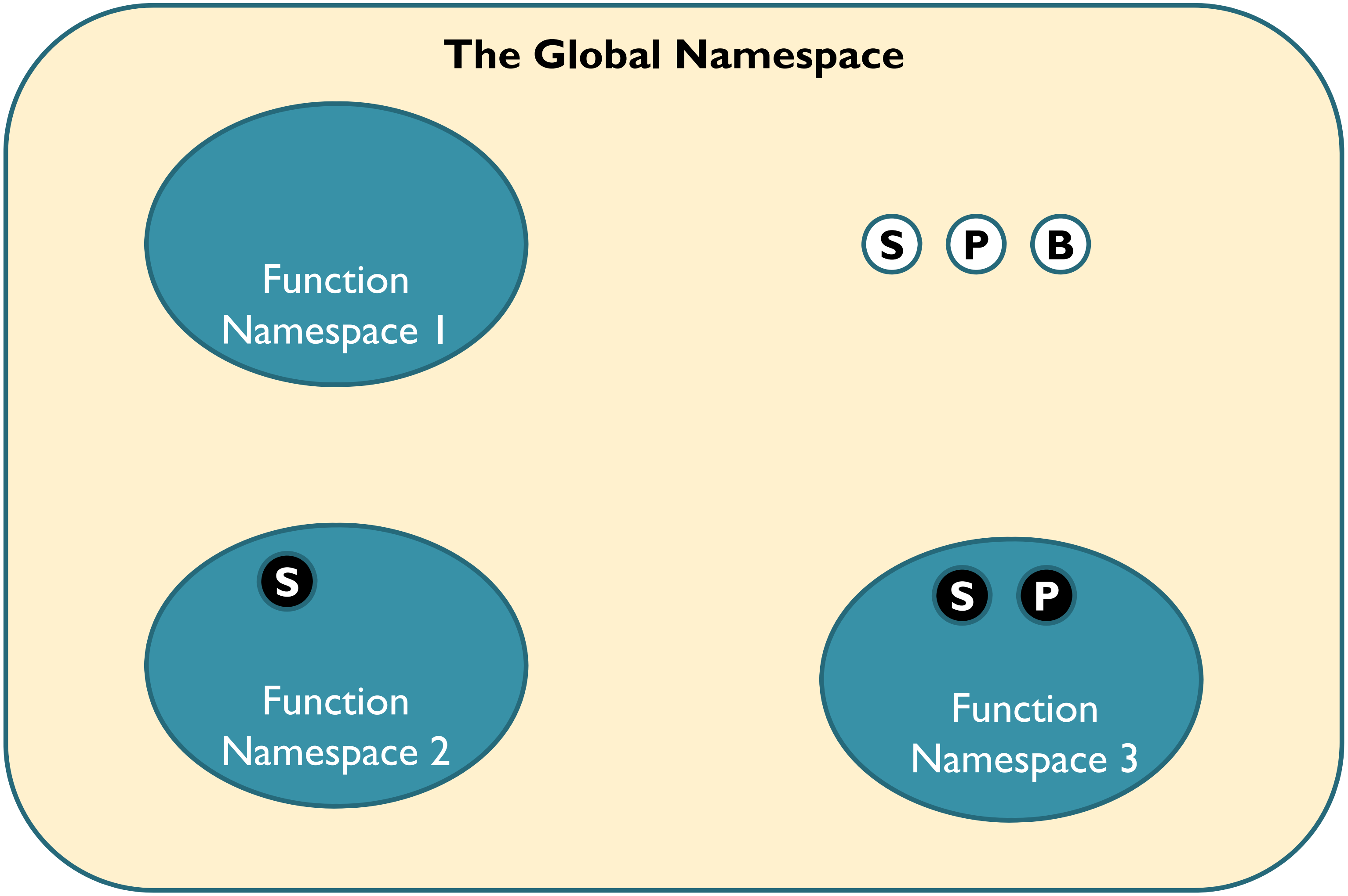
Functions and function variables Key Concepts



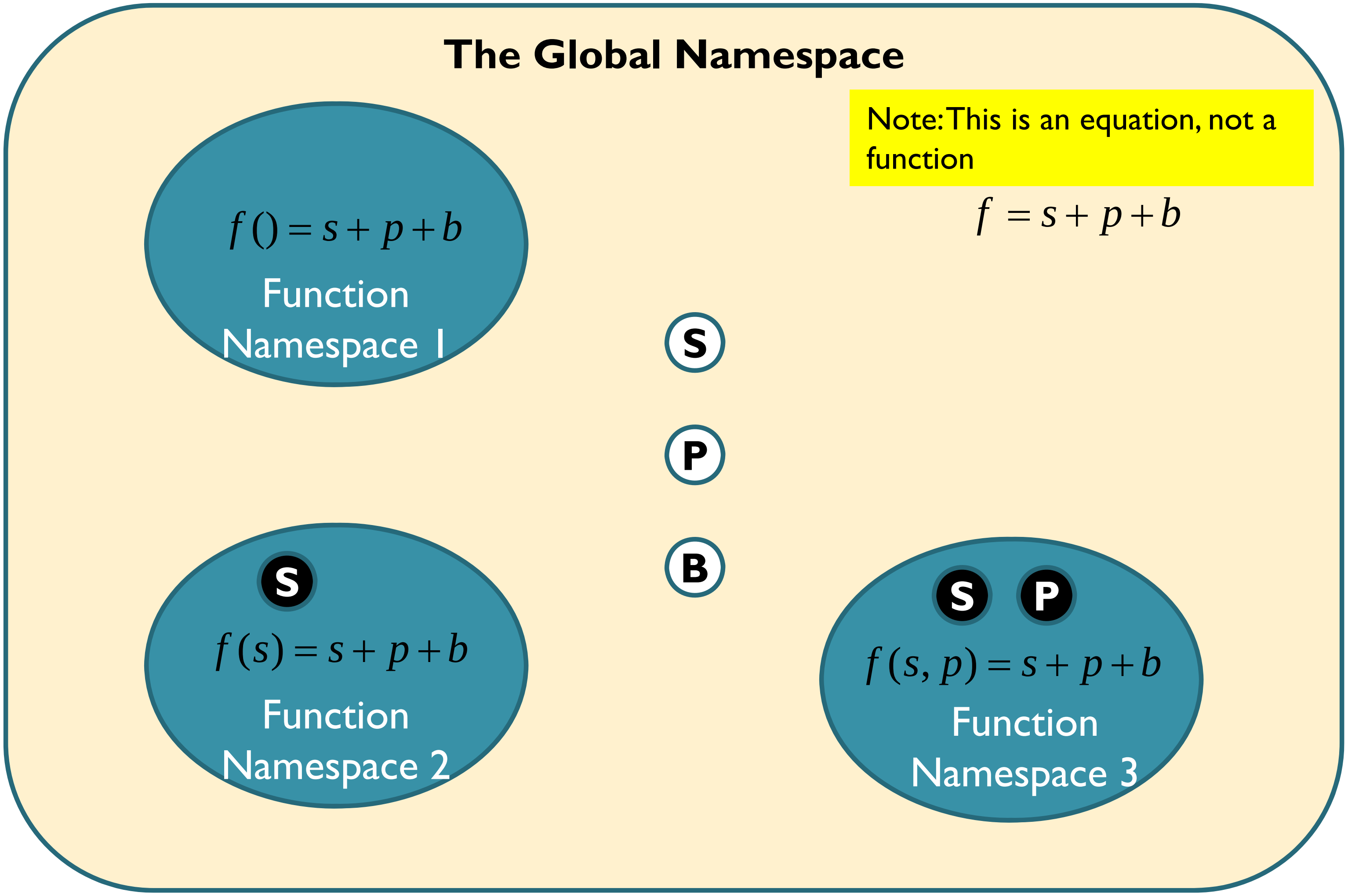
Functions and function variables Key Concepts



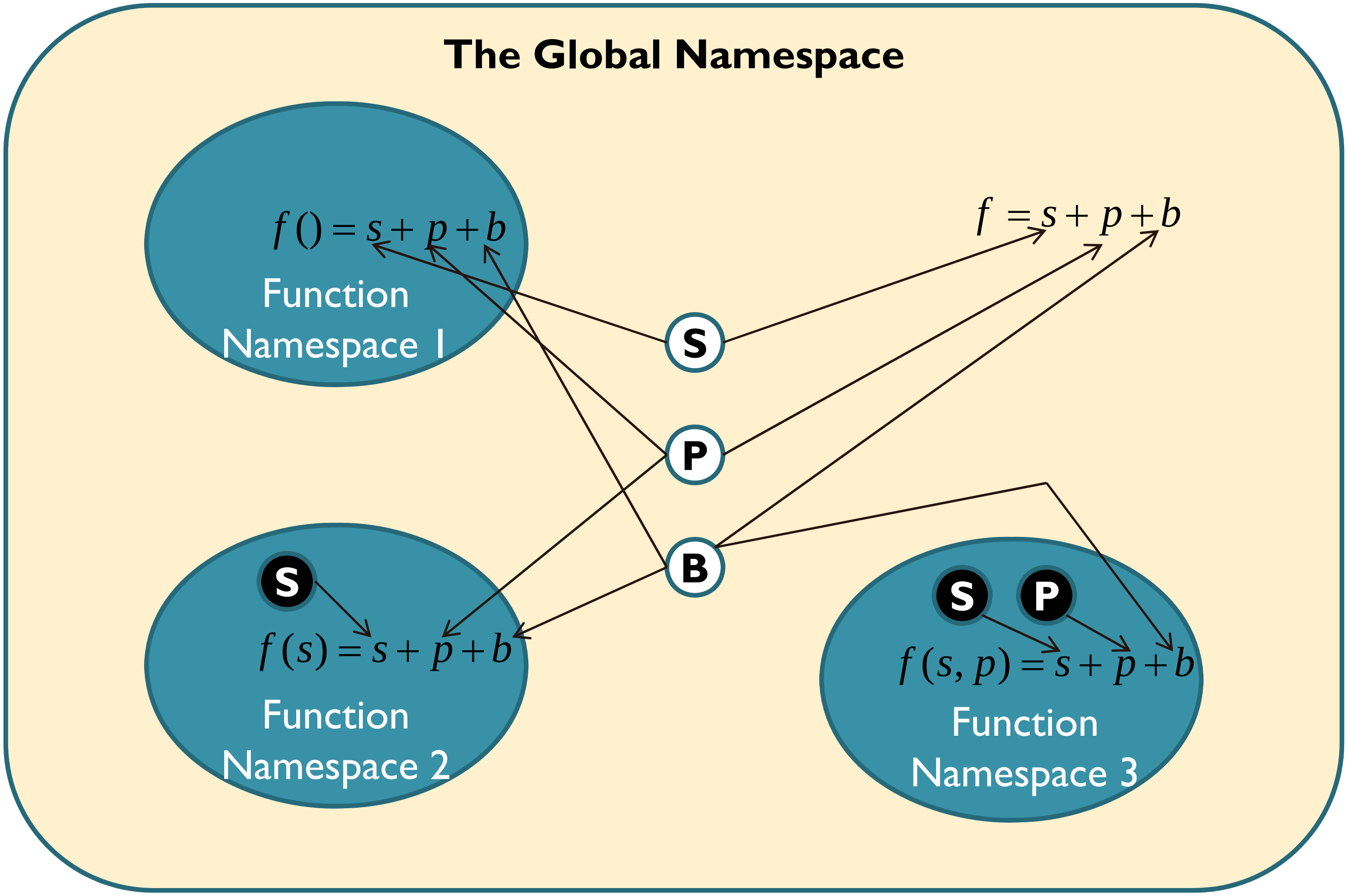
Functions and function variables Key Concepts



Functions and function variables
Key Concepts



Functions and function variables
Key Concepts



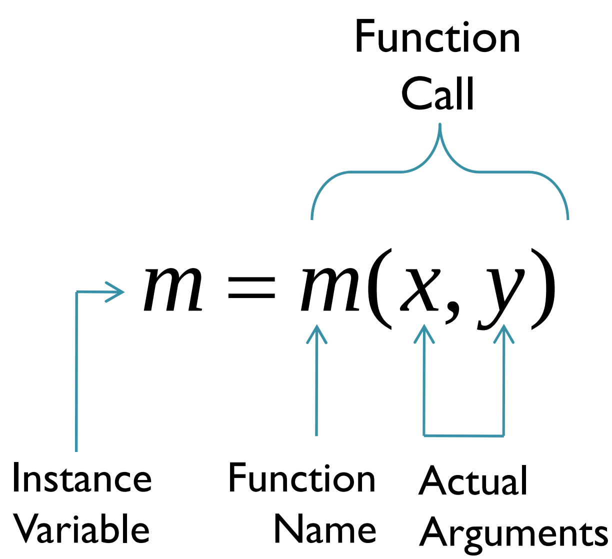
Functions and function variables Key Concepts

- Observations
  - A function with no parameters is functionally no different than an equation
  - Formal function definitions reduce confusion regarding local versus global variables, especially if the function uses variables of the same name as a global variable
  - If a function is defined with a certain number of "parameters," then it will also need to be invoked with the same number of "arguments."

Using a function

Invoking a function involves "calling" a function with specific "arguments" that the function will then use to produce a result that is set to the instance variable

$$m = m(x, y)$$



In order help prevent mistakes, I prefer to only show instance variables as part of an invocation, not as part of the function definition

Using a function

- Invoking or Calling: The act of using a function that is performed in multiple steps
  - Step 1: Pass the arguments to the function
  - Step 2: Replace the local variables with the arguments
  - Step 3: Set the instance variable to the evaluated function's result

Using a function

- **Informal Invocation:** Sometimes uses in mathematics, Step 1 looks like mathematical assignment statements:

$$s = 5$$
$$p = 3$$

In Einstein's 1905 derivation, he uses informal invocation

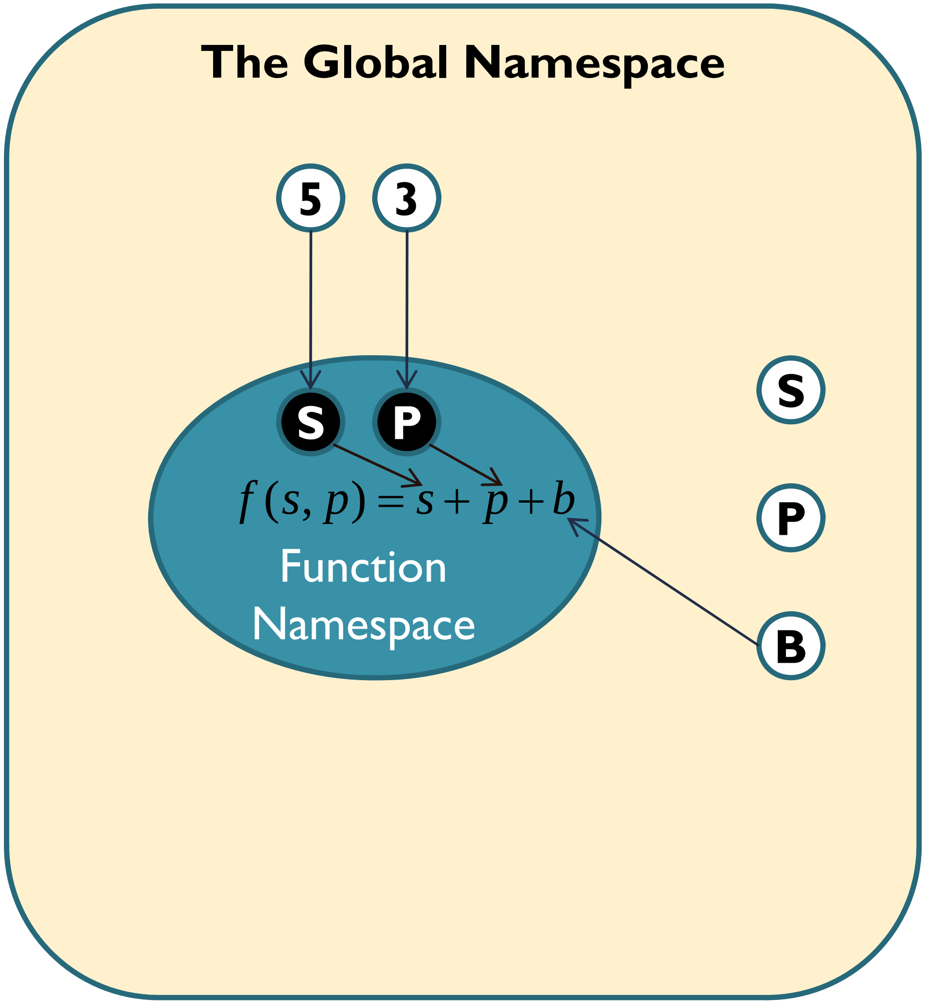
- **Standard Invocation:** Used in mathematics and computer science. Step 1 looks similar to a function signature, but uses the "real" values you intend to have the function use

$$f = f(5,3)$$

- **Formal Invocation:** Used in computer science. Step 1 looks the most complicated, but avoid the positional characteristic of the standard invocation

$$f = f(s = 5, p = 3)$$

Function Invocation - Key Concepts



The function:

$$f(s,p) = s + p + b$$

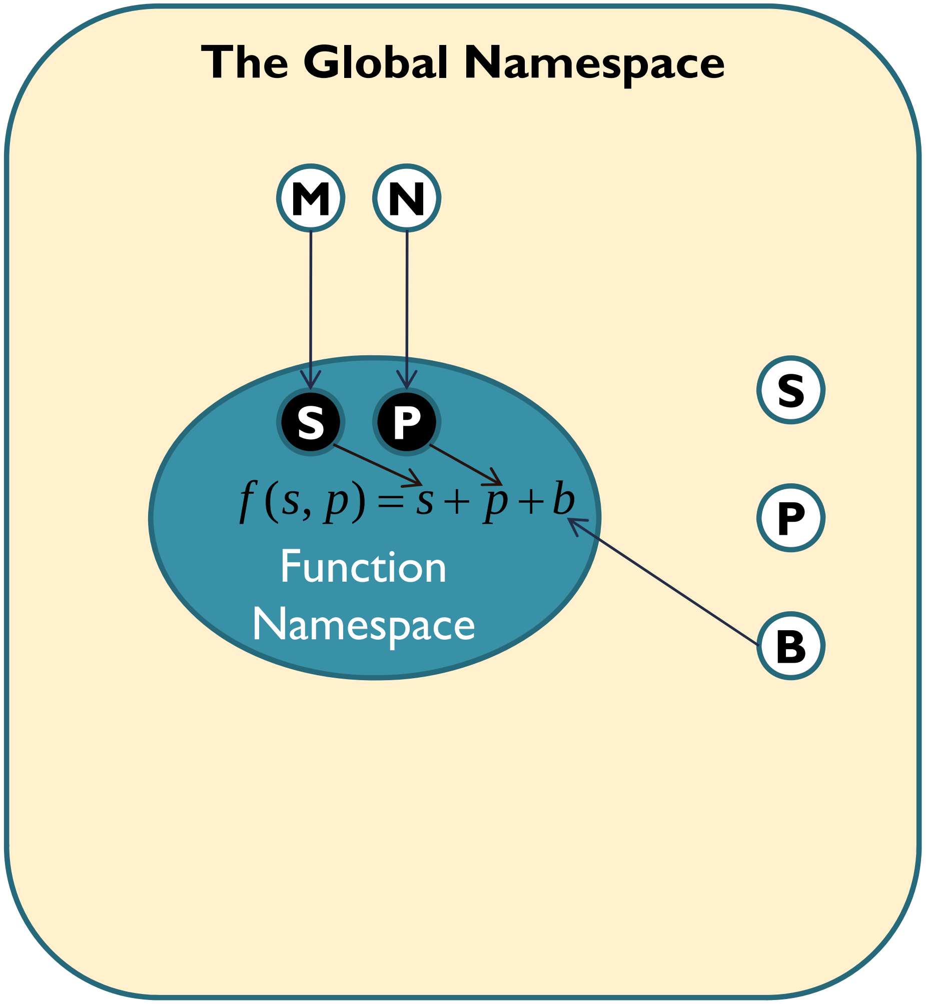
When invoked as:

$$f = f(5,3)$$

Is evaluated as:

$$f = 5 + 3 + b$$

Function Invocation - Key Concepts



The function:

$$f(s,p) = s + p + b$$

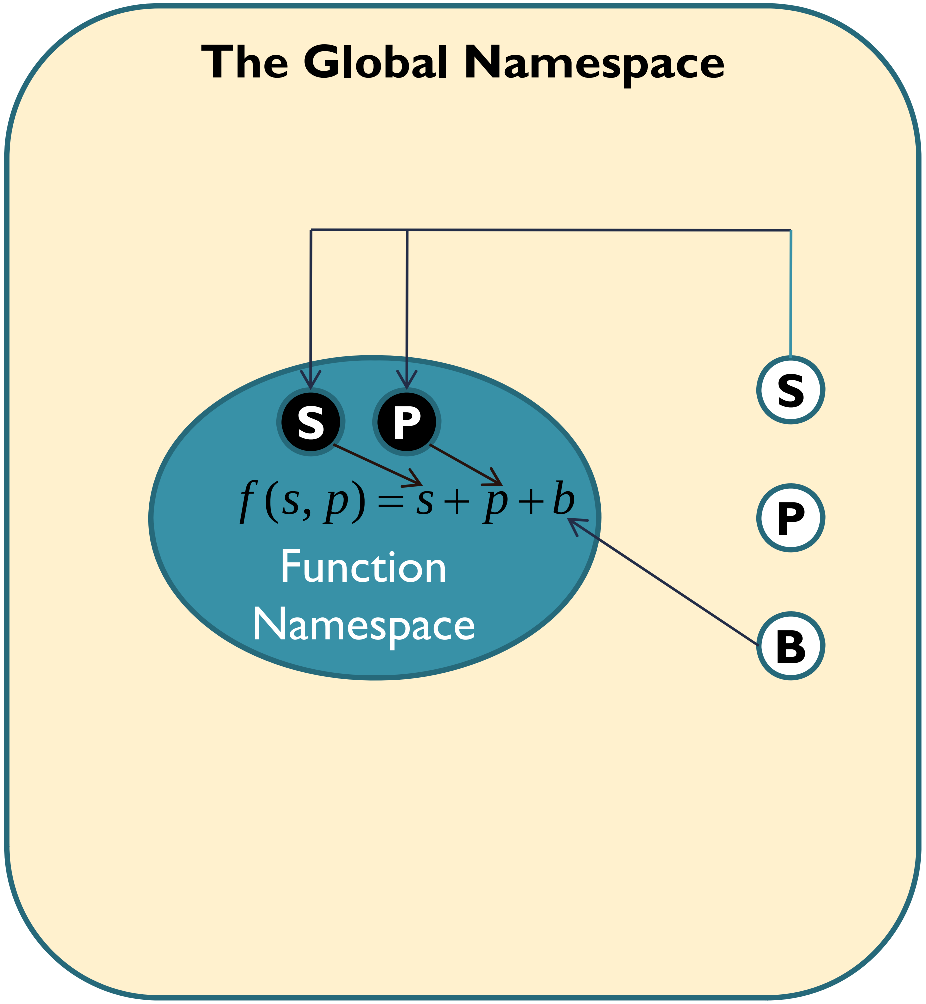
When invoked as:

$$f = f(m,n)$$

Is evaluated as:

$$f = m + n + b$$

Function Invocation - Key Concepts



The function:

$$f(s,p) = s + p + b$$

When invoked as:

$$f = f(s,s)$$

Is evaluated as:

$$f = s + s + b$$

Caution!

- Problems happen when you use arguments, during invocation, that have the same name as a function's local variables
- Be careful simplifying after invocation. Based on the previous slide, the following are all incorrect simplifications of the original function

$f = 2s + b$	*(Informal function specification)*
$f() = 2s + b$	*(Semi-formal function specification)*
$f(s) = 2s + b$	*(Formal function specification)*
$f(s, p) = 2s + b$	*(Formal function specification)*

- <u>CAUTION:</u> Notice that the instance equation can be simplified as: $f = 2s + b$ highlighting the need to formally specify whether f is a function or an instance equation
- <u>Local Variables are Place Holders:</u> Problems happen when you confuse global variables (or arguments) with local variables (for example, during simplification of a function)

Tau is function, not an equation

We know that Tau is a function

- Einstein calls it a linear "function"
- A function is the result of a Partial Differential Equation (PDE)
- In setting up his PDE, Einstein invokes the Tau function three times, each time he invokes it with four parameters

The Xi Derivation - Explained

When properly derived, Xi is simply c multiplied with a specific invocation of Tau.

$$\tau(x', t) = \alpha(t - \frac{vx'}{c^2 - v^2})$$

$$\xi = c * \tau(x - vt, (x - vt)/(c - v))$$

$$\xi = c * \alpha((x - vt)/(c - v) - v(x - vt)/(c^2 - v^2))$$

$$\xi = \alpha \frac{x - vt}{1 - \frac{v^2}{c^2}}$$

Invocation is consistent with Tau's usage in the original PDE

Einstein's Xi Derivation - Explained

- When viewed as an equation, when Einstein says in his 1905 paper that $t = x'/(c-v)$, this is an algebraic mistake since *t* cannot simultaneously be an independent and a dependent variable.

- However, when viewed as a function, when Einstein says in his 1905 paper that $t = x'/(c-v)$, he is simply using the informal invocation method to set the local variable *t* to the arguments $x'/(c-v)$, which is syntactically correct.

The Tau Derivation – Explained

When properly derived, Tau is invoked the same way as a as it was when it was used to derive Xi.

$$\tau(x',t)=\alpha(t-\frac{vx'}{c^2-v^2})$$

Invocation is consistent with Tau's usage in the original PDE and in the Xi invocation

$$\tau=\tau((x-vt),(x-vt)/(c-v))$$

$$\tau=\alpha((x-vt)/(c-v)-\frac{v(x-vt)}{c^2-v^2})$$

$$\tau=\frac{\alpha}{c}(\frac{x-vt}{1-\frac{v^2}{c^2}})$$

Einstein's Tau Derivation – Explained

Einstein's Tau simplification confuses the global variable *t* with the local variable *t*, producing an incorrect instance equation.

t is a local variable

The x' parameter is replaced with the argument x-vt during invocation; occurring when Einstein says replace x' with its value.

$$\tau(x',t) = \alpha\left(t - \frac{vx'}{c^2 - v^2}\right)$$

$$\tau = \tau(x - vt)$$

The *t* parameter is not replaced with an argument during invocation. Function usage is not consistent with the original PDE

Local Variable

$$\tau = \alpha\left(t - \frac{v(x - vt)}{c^2 - v^2}\right)$$

Global Variable

The local variable (or placeholder) variable *t* is incorrectly simplified with the global variable *t*.

$$\tau = \alpha\left(\frac{t - \frac{vx}{c^2}}{1 - \frac{v^2}{c^2}}\right)$$

Einstein's Tau Derivation - Explained

- Invoking and simplifying Tau with only one function argument disagrees with the number of parameters given in the original Tau Partial Differential Equation
- Syntactically, a local function variable (or placeholder variable) cannot be simplified with an argument variable (or a global variable) of the same name
- The problem can be resolved in one of two ways:
  - Use a different variable in the function definition (e.g., *t'*) as the local variable
  - Properly invoke the Tau function with both arguments (e.g., as we performed in deriving Xi)

Summary

- The rigorous treatment of functions, and their anatomy, is explained in modern disciplines such as Computer Science:
  - Made the discover of the problem possible and
  - Explains why this problem has been so elusive
- Functions explain the apparent paradox regarding:
  - How Xi, which appears to be incorrectly derived, is actually correct*
  - How Tau, which appears to be a straight forward simplification, is actually incorrectly simplified

** In its un-normalized form*

Appendix - Definitions

Definitions – At Function Definition

- **Function Definition:** The combination of the function signature and the function body.
- **Function Signature:** The combination of the function name and it parameters
- **Function Body:** The "equation" that makes up the function
- **Function Name:** The name of the function
- **Function Parameters:** Defines the local variables (or placeholders) that will be used by the function.
- **Global Variable(s):** Can be used everywhere. Variables used by a function that are not previously defined as a function's parameters
- **Local Variables:** Can only be used within the function. Within a function, local variables will override global variables of the same name

Definitions – At Function Invocation

- **Namespace:** Can be either local (within a function) or global. Defines where and how variables can be used.
- **Scope:** Defines where and how a variable can be used whether inside or outside of a function. *Scope* is from a variable's perspective and *namespaces* are from a function's or equations perspective.
- **Function Arguments:** The values that are "passed" to the function which are then used by the function, replacing the function's local variables
- **Instance Variable:** The resulting value following the invocation of a function. Instance Variables in the same namespace must have the same value (e.g., invoke the same function with the same arguments), otherwise it is a different instance variable (Note: Instance Variables can be considered Instance Equations if they require specific values in their variables before they can be evaluated)