Revisiting the Ives and Stillwell experiment: Comparing the accuracy of SRT against the model of Complete and Incomplete Coordinate Systems

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Special Relativity Theory (SRT) is one theory that accurately explains the results of the Ives and Stillwell atomic clock experiment. SRT has been challenged on mathematical grounds with the discovery of inconsistencies in the derivation of the transformation equations. The model of Complete and Incomplete Coordinate Systems (CICS) corrects the mathematical and theoretical problems with SRT and defines a set of equations for oscillating waves in moving systems. Here we show that the CICS model offers better predictive capabilities than SRT in evaluating the Ives and Stillwell experiment, as measured by the amount of error between the predicted and actual results. Importantly, this finding supports the use of one-half a wavelength in the CICS equations to account for the bi-directional nature of wavelength.

Introduction

The Ives and Stillwell atomic clock experiment[1] is often cited as experimental verification of Special Relativity Theory (SRT).[2] In fact, it has been suggested that other theories may be incapable of accurately predicting the results of the experiment.[3] However, SRT has been shown as mathematically inconsistent, with corrections established in the model of Complete and Incomplete Coordinate Systems (CICS).[4,5,6] Therefore, the CICS model must be shown to accurately predict the results of the Ives and Stillwell experiment.

The purpose of the Ives and Stillwell atomic clock experiment was to measure the shift in the length of one-half a wavelength and the Doppler displacement of a hydrogen atom in a contained canal ray tube with a wavelength of 4861 angstroms with experimental velocities of 0.5% the speed of light.[7] The goal was to experimentally demonstrate the Doppler effect as an indicator of time elongation.[8] A successful experiment would yield actual results close to the predicted values. While Ives and Stillwell were proponents of an alternative theory suggested by Larmor and Lorentz,[9] their equations appear similar to those asserted by SRT. Thus, advocates of SRT often cite this experiment as confirming the validity of SRT.

The experiment measured two results. The first experimental result was the measurement in the change in length of one-half a wavelength, referred to by Ives and Stillwell as the "shift in the center of gravity."[10] The second experimental result was the measurement of the Doppler shift.[11] Ives and Stillwell computed the displacement as the difference between the approaching and receding observations.[12] This paper will compare the predicted *shift* and Doppler *displacement* to show that the CICS model performs as well as SRT in predicting the actual *shift* results and performs slightly better than SRT in predicting the actual *displacement* results.

The Special Relativity Theory Predictions

In their analysis, Ives and Stillwell do not define specific SRT-based equations that are used to produce the expected *shift* and *displacement* results. In addition, such equations are not explicitly defined as part of SRT. However, these equations can be found by using accepted SRT-based equations as a foundation. The shift equation is based on Einstein's X-axis transformation equation. Thus, the equation that produces the expected shift in the center of wavelength is

$$shift = \frac{\lambda}{\sqrt{1-\frac{v^2}{c^2}}} - \lambda, \qquad (1)$$

where λ is the original wavelength used in the experiment, c is the speed of light, or 299,792,458 m/s, and v is the velocity, which is controlled by the experimenters. Interestingly, this equation represents a change in total wavelength, which differs from the change in one-half a wavelength (or change in the "center or gravity") measured by the experiment.

Because SRT does not appear to offer a stand-alone equation to compute the Doppler displacement, two alternatives are considered. Thus, the equations for computing the Doppler displacement are the standard Doppler displacement equation and the relativistic Doppler displacement equation, which are defined as

$$disp = \frac{\lambda}{\left[\frac{w}{w+v}\right]} - \lambda \, , \tag{2}$$

and

$$disp = \frac{\lambda}{\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}} - \lambda \, , \tag{3}$$

respectively, where λ is the original wavelength. The standard Doppler equation is generally applied to waves in general (e.g., sound) where $v << c$ and w is the velocity of the wave. The standard Doppler equation is sometimes viewed as a special case of the relativistic Doppler equation. The relativistic Doppler equation is generally applied to light and other phenomena traveling at relativistic speeds.

The Complete and Incomplete Coordinate Systems Model Predictions

The model of Complete and Incomplete Coordinate Systems introduces a set of equations that describe the behavior of oscillating waves in moving systems.[13] In addition to the moving coordinate systems found in SRT, the CICS model defines two specific types of systems; Complete and Incomplete.[14] The CICS model also reestablishes an electromagnetic wave medium or ether.[15] In a Complete Coordinate System, the wave medium is contained within, and travels with, the coordinate system.[16] In an Incomplete Coordinate System, the coordinate system travels through the wave medium which is not contained by the coordinate system.[17] The Ives and Stillwell experiment is an experiment on an Incomplete Coordinate System.

While SRT defines two types of transformations, fixed-point and wave-front, CICS establishes three.[18] It revises the fixed-point equations, extends the wave-front equations, and introduces a third set of equations that explain the behavior of oscillating waves in the moving coordinate system.[19] According to the CICS model, the length of an oscillation and the time to travel that length in an Incomplete Coordinate System elongate as a result of applied velocity. A key distinction between the CICS model and previous models, such as Lorentz or Einstein, is its recognition of the bi-directional nature of oscillating waves. Importantly, the CICS model defines x' in the equation

$$\xi = \frac{x'}{1 - \frac{v^2}{w^2}}, \tag{4}$$

as representing the length of one-half an oscillation.[20] In other words, x' represents one-half a wavelength.[*] The CICS model predicts the shift in length of one-half the wavelength as $\xi - x'$, or

$$shift = \frac{\frac{\lambda}{2}}{1 - \frac{v^2}{w^2}} - \frac{\lambda}{2}. \tag{5}$$

[*] Notice that x' can also represent the number of oscillations in one-half a wavelength when it is a measure of frequency (e.g., Hertz).

As defined by the CICS model, the displacement is the difference in length between the receding and approaching waves in the moving coordinate system. The equation is found by subtracting the receding wave equation, $\frac{x'}{w+v}$, from the approaching wave equation, $\frac{x'}{w-v}$, to find the time associated with the displacement.[21] This time is multiplied by the wave propagation velocity, w, since the displacement in the experiment is a measurement of length. Consistent with the CICS model, this difference in length is defined as

$$disp = \frac{2vw\frac{\lambda}{2}}{w^2 - v^2}. \qquad (6)$$

Discussion

The expected results of SRT and the CICS model are compared against the actual results of the experiment. Table I presents the analysis of the *shift* equations. Column 1 is the plate numbers as given in Ives and Stillwell's paper. Column 2 is the computed velocity, using the velocity equation presented in their paper. Column 3 presents the expected results of the CICS model as found using Eq. 5. Column 4 presents the expected results of SRT as found using Eq. 1. The actual results of the Ives and Stillwell experiment are given in Column 5.

Table I. Expected and Actual Results for *shift* in the "center of gravity".

(1) Plate	(2) Velocity	(3) CICS Shift	(4) SRT Shift	(5) Experimental Results Shift	(6) Difference between CICS and SRT predictions
169	638315.56	0.0110	0.0110	0.0110	1.25E-08
160	864655.47	0.0202	0.0202	0.0185	4.20E-08
163	949764.22	0.0244	0.0244	0.0225	6.12E-08
170	1016987.79	0.0280	0.0280	0.0270	8.05E-08
165	867739.12	0.0204	0.0204	0.0205	4.26E-08
172	1151434.93	0.0359	0.0359	0.0345	1.32E-07
172	933729.24	0.0236	0.0236	0.0215	5.72E-08
177	1317952.03	0.0470	0.0470	0.0470	2.27E-07

One interesting finding is that the CICS model and SRT predict essentially identical results for the shift in the "center of gravity." It is important to note that the CICS model and SRT predictions are not exact and their difference is given in Column 6. This difference is computed as

$$diff = \left[\frac{\frac{\lambda}{2}}{1-\frac{v^2}{w^2}} - \frac{\lambda}{2} \right] - \left[\frac{\lambda}{\sqrt{1-\frac{v^2}{c^2}}} - \lambda \right]. \tag{7}$$

The difference between the expected results of the CICS model and SRT increases as velocity increases. Given the degree of accuracy of the actual results at the velocities involved, the Ives and Stillwell experiment is unable to produce results that would enable one to differentiate between the two models.

Table II presents the analysis of the *displacement* equations. Column 2 presents the predictions of the SRT-based relativistic Doppler equation, Eq. 2. Column 3 presents the predictions of the standard Doppler equation, Eq. 3. Column 4 presents the predicted results of the CICS displacement equation, Eq. 6. Column 5 presents the actual results obtained from the experiment.

Table II. Expected and Actual results of the Doppler *displacement*.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Expected Displacement			Actual Results	Accuracy	
Plate	Relativistic Doppler	Standard Doppler	CICS	Observed Shift	Relativistic Doppler	CICS
169	10.3610	10.3500	10.3500	10.35	0.0110	0.0000
160	14.0403	14.0200	14.0201	14.02	0.0203	0.0001
163	15.4245	15.4000	15.4002	15.40	0.0245	0.0002
170	16.5181	16.4900	16.4902	16.49	0.0281	0.0002
165	14.0904	14.0700	14.0701	14.07	0.0204	0.0001
172	18.7060	18.6700	18.6703	18.67	0.0360	0.0003
172	15.1637	15.1400	15.1401	15.14	0.0237	0.0001
177	21.4172	21.3700	21.3704	21.37	0.0472	0.0004
mean	15.7151	15.6888	15.6889	15.69	0.0264	0.0002
stddev	3.3081	3.2971	3.2973	3.2971	0.0110	0.0001

As shown in Columns 6 and 7, the accuracy of the CICS model is 0.0002±0.0001, while the accuracy of the SRT based Relativistic Doppler equation is 0.0264±0.0110. The error required to align the predicted CICS expected results with the observed results is much smaller than the error required to align the SRT based Relativistic Doppler expected results with the observed results. In actuality, the amount of error between the CICS expected results and the experimental result is beyond the accuracy of the experimental measurement of two decimal places and is interpreted as an exact match. This finding suggests a better fit of the actual results of the *displacement* experiment with the predictions of the CICS model than with those of the relativistic Doppler displacement equation. The standard Doppler-based SRT displacement equation also produces better results than the relativistic Doppler-based SRT displacement equations. However, this is not surprising since the standard Doppler measurement was used to compute the velocity.

These findings also suggest that the Doppler effect can be computed using the CICS equation as

$$f' = f + \frac{2vw\frac{f}{2}}{w^2 - v^2}, \tag{8}$$

which simplifies as

$$f' = f(1 + \frac{vw}{w^2 - v^2}). \tag{9}$$

where f is the original frequency and f' is the computed Doppler frequency. The standard Doppler equation and the CICS equations, presented above, produce similar results, which distinguish themselves from one another as velocity increases. This finding, that a CICS-based equation can be used to compute Doppler shifts, will require further exploration.

Conclusion

As presented in Table III, This analysis has shown that the model of Complete and Incomplete Coordinate Systems accurately predicts the results of the Ives and Stillwell atomic clock experiment. It has shown that SRT and the CICS model both predict the same *shift* results and that the CICS model is better at predicting the *displacement* results, as determined by the amount of error between the expected and actual results.

Table III. Fitness evaluation of the Ives and Stillwell experiment evaluated against SRT and the CICS model.

	Model	
Evaluation Criteria	**CICS**	**SRT**
Actual results for the one-half wavelength (or "center of gravity") shift are consistent with the expected results produced by the model	Yes	Yes
Actual results for displacement observation shift consistent with the expected results produced by the model	Yes	No SRT does not formally define an equation to compute this result. The Relativistic Doppler shift equation is used instead.
Amount of experiment displacement error (average adjustment needed to have actual results and the model's expected results match exactly, measured as the expected result subtracted from the observed result)	0.0002	0.0264
Maximum error (adjustment needed to have actual results and model's expected results match exactly)	0.0004	0.0472
Actual results are interpreted as being consistent with the model	Yes	Yes

The actual results of the Ives and Stillwell experiment are qualitatively consistent with the mathematical predictions of both SRT and the CICS model. Thus, the Ives and Stillwell experimental results alone cannot be used to disqualify SRT or CICS as valid models. These findings support the validity of the CICS model in predicting the actual results of the Ives and Stillwell experiment. Importantly, the CICS model predicts the same *shift* results as SRT and appears to be significantly more accurate in predicting the *displacement* results. This analysis supports the use of the CICS equations which take into account the bi-directional nature of wavelength.

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[16] Ibid.

[17] Ibid.

[18] Ibid.

[19] Ibid.

[20] Ibid.

[21] S. Bryant, 2005, op. cit. (see reference 6)