A Brute-Force Mathematical Challenge to Special Relativity

Steven Bryant
Primitive Logic, Inc , 704 Sansome Street, San Francisco, CA 94111
eMail: Steven.Bryant.Email@RelativityChallenge.com
web: www.RelativityChallenge.com

Einstein's 1905 derivation of Special Relativity Theory (SRT) defines four transformation equations that are generally accepted as resulting from mathematically correct derivations. This paper mounts a brute-force challenge to Einstein's derivation and reveals a mathematical inconsistency in the SRT time transformation equation. Specifically, the brute-force analysis of the ξ (Xi) transformation reveals a specific algebraic problem in its derivation. The difficulty in identifying and communicating this finding is that the problem does not manifest itself in the validity of the ξ equation, but rather in the validity of the τ (Tau) equation. Once the root cause is identified and τ is corrected, the resulting system of equations will require that the theoretical underpinnings of SRT be re-examined.

Introduction

Einstein's Theory of Special Relativity (SRT) was published in 1905 [1] and has since become the foundational cornerstone of modern physics. In fact, R. Serway states that the "special theory of relativity alone represents one of the greatest intellectual achievements of all time." [2] Professional and amateur physicists have reviewed SRT critically for the past century. While many critiques have been offered to challenge SRT, most, if not all, have first introduced new variables, required the adoption of a new perspective, or asked the reader to accept the author's analysis of one of SRT's paradoxes. Acceptance of a new perspective, variables, or analysis of the paradoxes is often met with resistance if the source is not considered a senior representative of the accepted physics community. [3]

This paper differs from previous challenges in that it requires the reader to accept the validity of Einstein's 1905 derivation and does not require the reader to first accept a new perspective, set of variables, or explanation of the paradoxes. Instead, the mathematical problem in Einstein's 1905 paper is illustrated by placing the validity of Einstein's derivation in conflict with the accepted rules of algebraic substitution. Once the problem is revealed, this paper will then show why the problem does not show up as an incorrect ξ (Xi) transformation as would be naturally expected, but rather as an incorrect τ (Tau) transformation.

Understanding Einstein's Derivation

The mathematical cornerstones of SRT are Einstein's four transformation equations, which are derived in §3 of his 1905 paper [1] and are presented in Table 1. In mathematical terms, these equations take a set of input values, (x, y, z, t), and transforms them into a set of output values, $(\xi, \eta, \varsigma, \tau)$. Input values are those values you provide when using the equations, while output values are those found by using the transformations.

Table 1. Einstein's Transformation Equations with Input and Output Values

input values	transformation equations	output values
x	$\xi = (x - vt)\big/\sqrt{1 - v^2 / c^2}$	ξ
y	$\eta = y$	η
z	$\zeta = z$	ζ
t	$\tau = (t - vx / c^2)\big/\sqrt{1 - v^2 / c^2}$	τ

Proponents of SRT often discuss the equations in terms of reference frames or coordinate systems. While such discussion is necessary in understanding Einstein's theory, it is immaterial to understanding the mathematical validity of the derivation. While some challenges to SRT use the equations, as given in Table 1, to suggest a problem in Einstein's theory, such an analysis does not reveal the root cause. To reveal the root cause of the problem, the individual steps performed in deriving Einstein's equations must be re-examined.

In mathematical terms, Einstein creates the ξ transformation in four steps; a beginning statement followed by three algebraic substitutions. [1] This derivation is summarized in Table 2.

Table 2. Einstein's ξ Xi Derivation.

Summary of Einstein's 1905 ξ (Xi) Derivation

1. Begin with $\xi = c\tau$
2. Since $\tau = \alpha(t - vx'/(c^2 - v^2))$, substitute for τ to yield $\xi = c\,\alpha(t - vx'/(c^2 - v^2))$
3. Since $t = x'/(c - v)$, substitute for t and simplify to yield $\xi = \alpha \dfrac{x'c^2}{c^2 - v^2}$
4. Since $x' = x - vt$, substitute for x' and simplify to yield $\xi = \alpha \dfrac{x - vt}{1 - \dfrac{v^2}{c^2}}$

Although not explicitly commented upon by Einstein in his 1905 derivation, the resulting ξ equation, along with the other three transformations for η, ς, and τ, are each multiplied by

$$\sqrt{1 - v^2 / c^2}\ ,$$

to produce the final SRT equations previously given in Table 1. This paper discusses the equations in their "un-normalized" form, before they have been multiplied by $\sqrt{1 - v^2 / c^2}$. Each of the mathematical statements given in Table 2, with the exception of the final "yield" statement, is explicitly found in §3 of Einstein's 1905 paper. Use of this final "yield" statement in this paper, while implied in Einstein's work and not explicitly stated, does not diminish the analysis presented herein.

The mathematical summary of the ξ transformation derivation, given in Table 2, confirms that Einstein performed three algebraic substitutions on the original $\xi = c\tau$ equation. As such, each statement on the right-hand side of "$\xi =$" must produce the exact same result in order for the derivation to be mathematically valid. This establishes a string of four mathematical statements that must produce the same output value when given the same input values. In other words, each statement in Table 2 must produce the same result on the right-hand side of the equals sign in order to mathematical conclude that

$$\xi = \alpha \frac{x - vt}{1 - \frac{v^2}{c^2}}.$$

If any of the equations on the right-hand side of the equal sign produces a different result when given the same input values, this would represent a mathematical error since it would violate the rules of algebraic substitution. This provides a means of analyzing the validity of Einstein's derivation.

The Brute-Force Challenge

A brute-force challenge simply takes real input values and evaluates each step of Einstein's derivation, as given in Table 2, to determine if they produce the same result. The input values for this challenge are $x = 50$, $y = 0$, $z = 0$, $t = 10$, $v = 5$, and $\alpha = 1$. These values are used to evaluate each of the statements in Table 2. As given in Table 3, the brute-force challenge reveals that two of the statements produce 0 as a result, while the other two statements produce 10c, or 2,997,924,580. This brute-force analysis shows that the series of equivalent statements has been broken and represents a mathematical error in Einstein's ξ derivation.

Table 3. Brute-force Analysis of Einstein's Equivalent ξ Statements.

Evaluation of Einstein's 1905 ξ (Xi) Derivation

Equation	*Given* $x = 50, y = 0, z = 0, t = 10, v = 5, and\ \alpha = 1$	*Result*
$\xi = \alpha \frac{x - vt}{1 - \frac{v^2}{c^2}}$		$\xi = 0$
$\xi = \alpha \frac{x'c^2}{c^2 - v^2}$	*Since only unknown is* x': $x' = x - vt$	$\xi = 0$
$\xi = c\alpha(t - vx'/(c^2 - v^2))$	*Since only unknown is* x': $x' = x - vt$	$\xi = 10c$
$\xi = c\,\tau$	*Since only unknown is* τ: $\tau = \alpha(t - vx'/(c^2 - v^2))$	$\xi = 10c$

Since the brute-force analysis confirms that there is an algebraic problem with Einstein's ξ derivation, this problem must be corrected for the derivation to be mathematically sound. Interestingly, once the derivation is corrected, we will show how Einstein serendipitously arrived at the correct un-normalized ξ equation. This anomaly hides the fact that there is a problem in the derivation that must be corrected, one that does not manifest itself in the validity of ξ, but rather manifests itself as an incorrect τ transformation. This discovery is further masked because the τ transformation appears to result from a very straight forward simplification. This anomaly is explained by understanding the root-cause of the problem and is resolved by correcting the derivation.

Understanding and Correcting the Root Cause

The root cause of the problem can be traced to Einstein's mistreatment of τ as an equation, rather than as a function, during the ξ derivation and again during the τ simplification. [4] This mistreatment results in the locally-scoped t variable (as defined by the τ function) being confused with the globally-scoped t variable, as used by the $x' = x - vt$ equation. [4] Because τ is a function, these two t variables are not the same. Mathematically, both t variables occupy different namespaces and, therefore, are different variables, even though they have the same name. [4] This problem can be resolved in one of two ways. [4] First, the transformation can be derived and simplified as a function rather than as an algebraic equation. [4][5] Second, the equation can continue to be derived algebraically by renaming one of the t variables. [4] While either approach will correct the namespace problem, the second approach is discussed in the remainder of this paper.

One way of identifying the origin of the problem is to consider Einstein's statement in §3 of his 1905 paper that $x' = x - vt$. [1] In this statement, x' is dependent upon the independent values x and t. Specifically, Einstein sets up his partial differential equation by stating "If we place $x' = x - vt$, it is clear that a point at rest in the system k must have a system of values x', y, z, independent of time." [1] We have to consider this statement carefully to determine its validity. To illustrate this point, re-write this statement to illuminate how it can be interpreted in two alternative ways:

1. If we place $x' = x - vt$, it is clear that a point at rest in the system k must have a system of values x', y, z, independent of time [, where time is represented by t].

2. If we place $x' = x - vt$, it is clear that a point at rest in the system k must have a system of values x', y, z, independent of time [, where time is represented by t'].

The first statement is incorrect since we have already determined that x' is dependent upon both x and t. In this sentence, x' cannot be simultaneously independent on t and dependent on t. The second statement is valid since x' is independent of t'. Einstein's next sentence, "We first define τ as a function of x', y, z and t [,]" [1] indicates that he selected the first interpretation, which as previously stated, is not correct. In order to correct the derivation algebraically, change Einstein's statement such that it agrees with the second interpretation and is correctly stated: "We first define τ as a function of x', y, z and t'." Table 4 presents the derivation of the ξ equation after making this change and assumes a re-derivation of the Partial Differential Equation using t' instead of t.

Table 4. Revised ξ Xi derivation

Revised ξ (Xi) Derivation

1. Begin with $\xi = c\tau$
2. Since $\tau = \alpha(t' - vx'/(c^2 - v^2))$, substitute for τ to yield $\xi = c\,\alpha(t' - vx'/(c^2 - v^2))$
3. Since $t' = x'/(c - v)$, substitute for t' and simplify to yield $\xi = \alpha \dfrac{x'c^2}{c^2 - v^2}$
4. Since $x' = x - vt$, substitute for x' and simplify to yield $\xi = \alpha \dfrac{x - vt}{1 - \frac{v^2}{c^2}}$

The steps given in Table 4 are mathematically consistent since each step produces the same output value when given the same input values. It is very important to notice that this corrected derivation arrives at the exact same ξ transformation equation as originally found by Einstein. Because the corrected ξ derivation and Einstein's original ξ derivation both arrive at the same result, identification of a problem is extremely difficult and can only be found by examining the derivation itself. The reason that Einstein was able to arrive at the right equation is because he performed the third step in Table 2, the substitution of t with $t = x'/(c-v)$, before the fourth step, the substitution of x' with $x' = x - vt$. He would not have arrived at the same result had he performed the substitution in the reverse order. Einstein's order dependence occurs because t is first treated as a dependent variable and then as an independent variable. The revision uses $t' = x'/(c-v)$ such that t' is distinguished from t, enabling the derivation of the correct transformation equation regardless of the substitution order.

While ξ is now consistently derived, this system of equations produces a different τ function than originally found by Einstein. Since t' was used in correcting the derivation, τ as a linear function is corrected as

$$\tau = \alpha(t' - vx'/(c^2 - v^2)),$$

and the τ simplification [4][5] results in the equation

$$\tau = \frac{\alpha}{c}\left(\frac{x - vt}{1 - v^2/c^2}\right).$$

Implications

There are several mathematical and theoretical implications that result from this analysis.

1. Einstein's ξ derivation, as summarized mathematically in Table 2, establishes Einstein's transformation equation, $\xi = \alpha\left(\frac{x - vt}{1 - v^2 / c^2}\right)$, as a specific instance of the general $\xi = c\tau$ equation. This finding that, mathematically, the ξ fixed-point transformation equation is a specific instance of the $\xi = c\tau$ wave-front equation will require that the theoretical underpinning of SRT be reexamined.
2. The problem in Einstein's derivation can be confirmed by validating Einstein's un-normalized transformations equations using $\xi = c\tau$, yielding

$$\alpha \frac{x - vt}{1 - v^2 / c^2} \neq \alpha c\left(\frac{t - vx / c^2}{1 - v^2 / c^2}\right)$$

in most cases. The corrected equations are mathematically consistent with the original $\xi = c\tau$ statement, yielding

$$\alpha \frac{x - vt}{1 - v^2 / c^2} = c\left[\frac{\alpha}{c}\left(\frac{x - vt}{1 - v^2 / c^2}\right)\right]$$

in all cases.

Conclusion

Einstein's transformation equations are primarily derived algebraically in his 1905 paper. Since this derivation must adhere to the accepted rules of algebraic substitution, a brute-force analysis reveals that the algebraic rule is violated. However, identification of this problem is difficult because, while there is a problem in Einstein's derivation, he arrives at the correct ξ equation (in its un-normalized form). As can be seen from this analysis, identification of a problem in Einstein's equations is difficult to communicate because: 1) the problem can only be found during a re-examination of the ξ derivation, and 2) the problem is manifest in the τ equation, which is generally considered to be correct, based on a straight forward simplification. The root-cause of the problem is identified as the mistreatment of τ as an equation rather than as a function. The correction changes the τ transformation and this change will have significant implications on the theoretical characteristics and continued validity of SRT.

References

[1] Einstein, "*On the Electrodynamics of Moving Bodies*", Annalen der Physik **17**, 891 (1905)

[2] Serway and Jewett, *Physics for Scientiest and Engineers with Modern Physics*, 6th edition, Brook/Cole – Thompson Learning, 1245 (2004)

[3] Personal conversation with Dr. Cynthia Whitney, NPA Conference, 2007

[4] Bryant, RelavitiyChallenge.Com Podcast, Episode 8 "*Tau is a Function*", www.RelativityChallenge.com, 2007

[5] Bryant, "*Understanding and Correcting Einstein's 1905 Time Transformation*", accepted by Galillean Electrodynamics. Pre-publication version available at www.RelativityChallenge.com