Communicating Special Relativity Theory's Mathematical Inconsistencies

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Einstein's Special Relativity Theory is believed to be mathematically consistent. Here we find subtle and difficult to detect mathematical mistakes in each of Einstein's derivations. In his 1912 paper, the mistake is traced to the misuse of **set** algebra instead of **statement** algebra. Specifically, he uses the "=" relation on the Real set instead of the "=" relation on the Binary set, incorrectly establishing the equivalence of equations. Because the two "=" relations operate on different sets, they cannot be used interchangeably. In his 1905 paper, he begins the derivation of the transformations with the equation $\xi = c\tau$. Mathematically, his final transformation equations fail the internal validity check because the stand-alone equation he derives for τ does not always equal $\tau = \frac{\xi}{c}$. This mistake is traced to the mistreatment of the time *function*, which is the result of the partial differential equation, as an *equation* rather than as a *function.*

Einstein's Special Relativity Theory (SRT) is well-confirmed experimentally and is generally accepted as internally consistent.[1] It is generally accepted that "any attempt to modify or disprove [SRT] has to rely upon either the quantitative predictions of different experimental results or the discovery of an internal logical inconsistency."[2] Quantitative results based on experimental challenges[3,4] can only separate theories into two classes, those that are consistent with the results and those that are not. It is unlikely that an experiment will invalidate SRT without the establishment of a new theory that is consistent with the existing results while being mathematically distinguishable from SRT in its quantitative predictions.

Logical consistency challenges to SRT[5,6] are largely based on the analysis of the implications and theoretical predictions associated with the SRT equations (e.g., time dilation, moving clocks and twins). These challenges, while initially convincing, have not disproved SRT since the paradoxes and implications have already been explained by the scientific community.[7]

Logical consistency challenges can also be based on the mathematical consistency of the derivations. Mathematical challenges to SRT are infrequent because of the amount of scrutiny placed on the theory[8] and the general acceptance that SRT is mathematically correct.[9] However, a mathematical challenge stands the greatest chance of success because of its precision. Mathematical arguments are unambiguous and "always stick to the precise mathematical definition, regardless of any colloquial usage."[10] The scope of this paper is to communicate mathematical inconsistencies in each of Einstein's SRT equation derivations.

Success Criteria

Success criteria must be established for a mathematical challenge to prove successful. Therefore, the following criteria are suggested.

1. The steps performed mathematically outweigh any supporting written text describing those steps. In other words, the challenge must be mathematical in nature and cannot be based on non-mathematical meanings associated with the equations or terms.
2. The steps must violate existing mathematical rules or fail to adhere to internal validity checks.

With the success criteria defined, we now show the inconsistencies in Einstein's derivations.

Einstein's 1905 Derivation

Consider the equation $a = c * b$, where c is a constant and a and b are variables. Importantly, and mathematically, if a is known and was derived as the equation $a = c * b$, b can always be found by dividing a by the constant c (e.g., $b = \frac{a}{c}$). This provides a means to check for internal consistency. Specifically, if we divide a by the constant c to independently arrive at b, and find that the result is different than if we simply used an equation to arrived at b, we have found a mathematical error since b cannot simultaneously have two different values.

Consider the steps Einstein performs in creating the transformation equations. In §3 of his 1905 manuscript, he states that[11]

$$\xi = c\tau \,. \qquad \text{Eq. 1}$$

He begins with the equation, $\xi = c\tau$, and since he earlier found that $\tau = \alpha(t - \frac{vx'}{c^2 - v^2})$, he replaces τ with the expression $\alpha(t - \frac{vx'}{c^2 - v^2})$, producing

$$\xi = c\tau = c\alpha(t - \frac{vx'}{c^2 - v^2})\,. \qquad \text{Eq. 2}$$

Einstein then states that $t = \frac{x'}{c - v}$ and replaces t with the expression $\frac{x'}{c - v}$, resulting in

$$\xi = c\tau = c\alpha(\frac{x'}{c - v} - \frac{vx'}{c^2 - v^2})\,, \qquad \text{Eq. 3}$$

which Einstein simplifies to

$$\xi = c\tau = \alpha \frac{c^2 x'}{c^2 - v^2} \qquad \text{Eq. 4}$$

or

$$\xi = c\tau = \alpha \frac{x'}{1 - \frac{v^2}{c^2}}\,. \qquad \text{Eq. 5}$$

Einstein's derivation of the ξ equation is presented in Figure 1.

Mit Hilfe dieses Resultates ist es leicht, die Größen ξ, η, ζ zu ermitteln, indem man durch Gleichungen ausdrückt, daß sich das Licht (wie das Prinzip der Konstanz der Lichtgeschwindigkeit in Verbindung mit dem Relativitätsprinzip verlangt) auch im bewegten System gemessen mit der Geschwindigkeit V fortpflanzt. Für einen zur Zeit $\tau = 0$ in Richtung der wachsenden ξ ausgesandten Lichtstrahl gilt:

$$\xi = V\tau,$$

oder

$$\xi = a\,V\left(t - \frac{v}{V^2 - v^2}\,x'\right).$$

Nun bewegt sich aber der Lichtstrahl relativ zum Anfangspunkt von k im ruhenden System gemessen mit der Geschwindigkeit $V - v$, so daß gilt:

$$\frac{x'}{V - v} = t.$$

Setzen wir diesen Wert von t in die Gleichung für ξ ein, so erhalten wir:

$$\xi = a\,\frac{V^2}{V^2 - v^2}\,x'.$$

Source: Annalen der Physik **17**, 891 (1905)

FIG 1. Einstein begins with the equation $\xi = c\tau$ as the foundation for deriving his transformation equations, resulting in $\xi = c\tau = \alpha \dfrac{c^2 x'}{c^2 - v^2}$.

Yet, in creating the stand-alone τ equation, he only simplifies $\alpha(t - \dfrac{vx'}{c^2 - v^2})$ by replacing x' with $x - vt$ to produce

$$\tau = \alpha \frac{t - \dfrac{vx}{c^2}}{1 - \dfrac{v^2}{c^2}} \qquad \text{Eq. 6}$$

Notably, in producing the τ equation, he does not first replace t with the expression $\frac{x'}{c-v}$. Einstein completes his ξ derivation by replacing x' with $x-vt$, and then multiplies all of the equations by $\sqrt{1-\frac{v^2}{c^2}}$ to produce the normalized equations

$$\xi = c\tau = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \text{Eq. 7}$$

and

$$\tau = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}. \qquad \text{Eq. 8}$$

Einstein provides a means to test the validity of the equations by using the equation $\xi = c\tau$. We have found that $\xi = c\tau = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$ and $\tau = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$. While $\frac{\xi}{c}$ should always equal τ, we find that generally[12] $\frac{x-vt}{c\sqrt{1-\frac{v^2}{c^2}}} \neq \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$. Thus, Einstein's 1905 derivation is not mathematically consistent using the rules of modern algebra.

The root cause of the problem in Einstein's 1905 derivation is the mistreatment of $\tau = \alpha(t - \frac{vx'}{c^2 - v^2})$ as an equation rather than as a function.[13] In fact, the author has shown that Einstein's time transformation, $\tau = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$, is incorrect[14] and should be calculated as[*] $\tau = \frac{x - vt}{c\sqrt{1 - \frac{v^2}{c^2}}}$. This error might have been discovered sooner had Einstein written the "*linear* function" as $\tau(x', y, z, t) = \alpha(t - \frac{vx'}{c^2 - v^2})$, confirming τ as a function.

Readers familiar with Special Relativity may want to associate the Greek variables ξ and τ with a wave-front and not with fixed point transformations. This paper's finding, while contrary to the accepted understanding, implies that the transformation equations are specific instances of the wave-front equations. This important exploration is beyond the scope of this paper, but is explained in *Reexamining Special Relativity*[15] and in *Understanding and Correcting Einstein's 1905 Time Transformation.*[16]

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[*] This equation is more properly written as $\tau = \alpha \frac{x - vt}{c\left[1 - \frac{v^2}{c^2}\right]}$ or $\tau = \alpha \frac{x'}{c\left[1 - \frac{v^2}{c^2}\right]}$.

Einstein's 1912 Derivation

Consider the equations

$$a = b$$
$$c = d. \qquad \text{Eq. 9}$$

Now consider these equations rewritten as

$$a - b = 0$$
$$c - d = 0. \qquad \text{Eq. 10}$$

It is easy to mathematically show that Equations 10 are mathematically equivalent to Equations 9. In other words, any solution that satisfies Equations 10 will also satisfy Equations 9, and visa-versa. What happens if we associate Equations 10 with one another, to produce

$$a - b = c - d\ ? \qquad \text{Eq. 11}$$

While at first appearance, Equations 10 can be combined using the transitive relation[†], it is mathematically incorrect for us to state that Equation 11 is an expression of the equivalence of the two statements comprising Equations 10.

The equivalence of Equations 10 is mathematically expressed using **statement** algebra, as

$$a - b = 0 \Leftrightarrow c - d = 0. \qquad \text{Eq. 12}$$

[†] Technically, Equations 10 cannot be associated with one another using the transitive relation. The transitive relation definition is $\forall x, y, z \in A, if\ \ x = y\ and\ \ y = z\ then\ x = z$. Since 0 is a constant in Equations 10, it cannot take on all values in the set and therefore does not satisfy the conditions for the transitive relation.

The use of the "=" relation in Equation 11, instead of the "$\Leftrightarrow$" operator, to associate Equations 10 with one another requires us to explore **set** algebra, specifically equivalence relations. Since statements, as expressed by Equations 9 and 10, are either **True** or **False**, they are members of the Binary set. This allows us to use the equivalence relation "=" for the Binary set. In this case, the "=" relation associates $a-b=0$ with $c-d=0$ such that

$$(a-b=0)=(c-d=0)\,. \qquad \text{Eq. 13}$$

Mathematically, "=" is the operator and it takes two Binary operands, resulting in the finite relation set {(**True, True**), (**False, False**)}. There is an infinite solution set for the variables a, b, c, and d that make this relation possible. The information that $a-b$ must equal 0 and that $c-b$ must equal 0 is contained within the "=" relation set on the Binary set.

Equation 11 also uses the "=" relation to create a statement. However, the "=" operator takes Real operands, not Binary operands. (Compare "$a-b$" with a Real result versus "$a-b=0$" with a Binary result). Thus, the resulting infinite relation set is $\{(x,x) \mid \forall x \in \text{Real}\}$. There is an infinite solution set for the variables a, b, c, and d that make this relation possible. The information that $a-b$ must equal 0 and that $c-b$ must equal 0 is never established. Because the set defined by the "=" relation in Equation 11 is infinite and operates on the Real set, it cannot be used as a replacement for the set defined by the "=" relation used in Equation 13 that is finite and operates on the Binary set. Thus, Equation 11 does not expresses the equivalence of $a-b=0$ with $c-d=0$.[‡]

Finally, we must determine whether this analysis changes if Equation 11 is written as

$$a-b=l(c-d) \qquad \text{Eq. 14}$$

where l is a generalized multiplier. The introduction of the generalized multiplier does not change the result since this statement still requires the "=" relation to operate on the Real set instead of the Binary set.

We validate this finding by checking if Equation 12 is equivalent to Equations 14 such that

$$[a-b=0 \Leftrightarrow c-d=0] \Leftrightarrow [a-b=l(c-d)]\,. \qquad \text{Eq. 15}$$

All solutions for the variables a, b, c, and d in Equation 12 are solutions to equations 14, where is l a generalized multiplier that is able to take on all Real values. However, the

[‡] In order to better illuminate this mathematical finding, readers with a programming background that includes C++ should consider that the following two overloaded operator== methods would not be confused with one another. Assume a Binary (a.k.a. Boolean) type and the overloaded AddToRelationSet methods exist.

```
Binary operator==( Binary op1, Binary op2 ) { if (op1==op2) AddToRelationSet( op1, op2 ); return op1==op2; }
Binary operator==( Real op1, Real op2 ) { if (op1==op2) AddToRelationSet( op1, op2 ); return op1==op2; }
```

Both methods may at first appear equivalent to one another because each tests two operands, adds them to a set if they aren't already members, and returns a Binary result. However, the two methods are not equivalent. The compiler uses the operand type of the invoking statement to correctly identify which one of the operator== methods would be called. The set created from the operator== taking Binary arguments would contain, at most, two entries, while the set created from the operator== taking Real arguments would not have an upper boundary on the number of entries. It is a programming error to use one of these sets as a replacement for the other.

reverse is not true. All solutions for the variables a, b, c, d and l in Equations 14 are not solutions to Equations 12. Consider the case where l is 0, which is a required condition if the equations are equivalent. As presented in Equation 15, equivalence requires that Equation 12 imply Equations 14, and visa-versa. Since Equations 14 does not always imply Equations 12 (e.g., $l = 0$, $a = 10$, $b = 10$, $c = 10$, and $d = 11$), the statements are not equivalent. Thus, Equation 15 is False, and the use of Equation 14 to express the equivalence of Equations 12 is a mathematical mistake.[§]

In his 1912 manuscript, Einstein begins his derivation by stating that[17]

$$\begin{aligned} x^2 + y^2 + z^2 &= c^2 t^2 \\ x'^2 + y'^2 + z'^2 &= c^2 t'^2 . \end{aligned} \qquad \text{Eq. 16}$$

These equations are in the form $a = b$ and $c = d$. He then rewrites them to produce[18]

$$\begin{aligned} x^2 + y^2 + z^2 - c^2 t^2 &= 0 \\ x'^2 + y'^2 + z'^2 - c^2 t'^2 &= 0 \end{aligned} \qquad \text{Eq. 17}$$

which are in the form $a - b = 0$ and $c - d = 0$. Einstein combines these equations to produce[19]

$$\lambda^2 (x^2 + y^2 + z^2 - c^2 t^2) = x'^2 + y'^2 + z'^2 - c^2 t'^2 , \qquad \text{Eq. 18}$$

which he states expressed the equivalence of the two statements comprising Equations 17.[20] Equation 18 is a mathematically incorrect statement for the equivalence of

[§] Since we have already shown that Equation 12 is equivalent to Equation 10, then in a similar manner, Equation 14 is not equivalent to Equation 10.

Equations 17 based on our previous analysis. The equivalence of Equations 17 can be expressed using statement algebra as

$$x^2 + y^2 + z^2 - c^2t^2 = 0 \Leftrightarrow x'^2 + y'^2 + z'^2 - c^2t'^2 = 0 \qquad \text{Eq. 19}$$

or using set algebra as

$$(x^2 + y^2 + z^2 - c^2t^2 = 0) = (x'^2 + y'^2 + z'^2 - c^2t'^2 = 0)\,. \qquad \text{Eq. 20}$$

Importantly, Equation 18 can be produced by using the transitive relation on

$$\begin{aligned} \lambda^2(x^2 + y^2 + z^2 - c^2t^2) &= g \\ x'^2 + y'^2 + z'^2 - c^2t'^2 &= g \end{aligned} \qquad \text{Eq. 21}$$

which are not the equations for spheres, except of course in the special case where g is 0 and λ^2 is 1. Equations 21 have a larger solution set than Equations 17.

Einstein continues his derivation to find a solution for Equations 18 using his Minkowski-based matrix.[21] While his matrix computation may be correct, Einstein's 1912 derivation is mathematically inconsistent because he has already violated the mathematical equivalence rules.

Einstein's *Relativity* Book Derivation

Einstein begins the derivation in Appendix 1 of his *Relativity* book by stating that[22]

$$\begin{aligned} x &= ct \\ x' &= ct'. \end{aligned} \qquad \text{Eq. 22}$$

As with his 1912 derivation, these equations are in the form $a = b$ and $c = d$. He then finds[23]

$$\begin{aligned} x - ct &= 0 \\ x' - ct' &= 0 \end{aligned} \qquad \text{Eq. 23}$$

which are in the form $a - b = 0$ and $c - d = 0$. Equations 22 and 23 are equivalent. He then mathematically concludes that[24]

$$\lambda(x - ct) = x' - ct' \qquad \text{Eq. 24}$$

expresses the equivalence of the two statements comprising Equation 23. Again, using the previous analysis, we have shown that this is a mathematically incorrect statement of the Equivalence of Equation 23. The equivalence of Equations 23 can be expressed using statement algebra as

$$x - ct = 0 \Leftrightarrow x' - ct' = 0 \qquad \text{Eq. 25}$$

or using set algebra as

$$(x - ct = 0) = (x' - ct' = 0). \qquad \text{Eq. 26}$$

Equation 24 can be produced by using the transitive relation on

$$\begin{aligned} \lambda(x - ct) &= g \\ x' - ct' &= g, \end{aligned} \qquad \text{Eq. 27}$$

which has a larger solution set than Equations 23. Equation 27 is equivalent to Equations 23 only in the special case where g is 0 and λ is 1. As in his 1912 manuscript, his entire derivation is invalidated because he violated the equivalence rules.

Conclusion

The goal of this paper was to highlight and communicate the mathematical inconsistencies in Einstein's derivations. We have analyzed Einstein's derivations to reveal mathematical errors in each. Each challenge satisfies the success criteria since they are not based on the colloquial meaning of any variable or equation and can be mathematically verified.

While Einstein's equations provide predictions that are sufficiently close to the experimental results,[25] the theory itself is challenged on mathematical consistency grounds. No amount of experimental evidence can make an internally inconsistent theory correct, no matter how "close" the theory may be in its predictive characteristics. A complete analysis of the mathematical correction, the meaning of the corrected equations, and the relationship of the corrected equations to existing experimental results are beyond the scope of this paper and are explained in *Reexamining Special Relativity*. Importantly, the author's alternative theory, as presented in *Reexamining Special Relativity*, remains consistent with the experimental results while being mathematically distinguishable from SRT.

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