The Twin Paradox: Why it is Required by Relativity

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The Twin Paradox is one of the most well known and debated paradoxes associated with Relativity theory. Opponents challenge Relativity theory on the grounds that the Twin Paradox reveals an underlying flaw in the theory. Such opponents feel that the existence of a paradox, in and of itself, is sufficient to disqualify the theory. Supporters explain the paradox by introducing the concept of acceleration into the theory, thus limiting the interpretation to the twin that was undergoing the force of acceleration. However, both interpretations fail to explain why Relativity requires the paradox, which is actually the result of using a length based model to interpret wavelength based observations. Here we show the proper use and interpretation of wavelength based observations using wavelength based equations, and how the mistaken use of length based equations results in time dilation, length contraction, and the Twin Paradox.

1. Introduction

Some challengers of Relativity theory have attacked its validity on the basis of the Twin Paradox [1, 2]. Relativity theory, which defines space-time concepts such as time dilation and length contraction, establishes that an object traveling at a faster velocity will experience a slower passage of time than an object traveling at a slower velocity [3]. The Twin Paradox identifies a specific problem associated with the reflective nature of how time dilation is defined and interpreted. The paradox begins with an assumption of two twin sisters, who are living on earth and, of course, are the same age. One sister is then placed into a rocket and propelled into space at nearly the speed of light. After 50 years, she returns to earth. From the perspective of the twin on earth, it is the sister in the rocket that was moving away, and as a result was undergoing time dilation and aging more slowly than her earthly bound sibling. However, from the perspective of the twin in the rocket, it is the sister on earth that was moving away and undergoing time dilation, resulting in the sibling on Earth aging more slowly. Some believe that the reflective nature of time dilation that results in the Twin Paradox is, in and of itself, sufficient to nullify Relativity theory. The author has previously established that such paradoxical arguments are inherently weak due to the fact that they assume that all of the preceding underlying assumptions, mathematics, and logic are correct [4].

Supporters of Relativity theory have offered explanations regarding their interpretation of time dilation that avoids production of a Twin Paradox [5]. However, there does not appear to be a single universally accepted explanation for the Twin Paradox [2]. A widely supported position is that time dilation occurs only with respect to the sister who undergoes the force associated with acceleration; in other words, the sister in the rocket. While this explanation is sufficient to neutralize the reflective nature of time dilation and remove the paradox, it is built upon information not present in Einstein's foundational papers [3, 6]. Rather, it is built upon an assumed relationship between acceleration and time. However, Einstein does not discuss acceleration as one of the variables in the development of his theory, so such a conclusion is interpretive rather than derived. While the acceleration explanation may be an acceptable answer to some, it is not acceptable to others. The acceleration explanation does not answer the question of why time dilation, length contraction, or the Twin Paradox are required artifacts of Relativity theory.

We have previously identified mathematical equations that provide more accurate predictions of experimental results than Relativity theory [7, 8]. This increased accuracy is defined as a smaller error between the predicted result and the actual result in experiments such as Michelson and Morley (error of <3 km/s versus 5 to 8 km/s), Miller (error of <1 km/s versus 9 to 11 km/s), and Ives-Stillwell (error of 0.001 Hz versus 0.02 Hz) [7, 8]. The revised mathematical equations that yield the increased accuracy are grounded in the finding that the experimental observations fall into one of two categories: Length based and Wavelength based observations. Because the revised analyses use wavelength based algorithms to interpret wavelength based experiments, they perform quantitatively better than the equations associated with Relativity theory. In this paper, we show that the inability of Relativity theory to properly distinguish wavelength from length creates its need for concepts like time dilation and length contraction, the former of which leads to the Twin Paradox. The introduction of a theoretical class that distinguishes between wavelength based and length based observations and equations leads to an easier to understand and more comprehensive theoretical model [9].

2. Discussion

One of the most important equations in physics establishes the relationship between Length, Velocity, and Time,

$$d = vt \tag{1}$$

We will name this the *Length equation*. Length and Time are both Measures (e.g., meters, seconds), while Velocity is a Rate (e.g., meters per second). This equation is the foundation for other derived equations in both Classical Mechanics and in Modern Physics. Maxwell, in his foundation work in the late 1800s, defined many terms, including length and time, which he then used in his work [11, 12]. These terms and equations found their way into the body of knowledge associated with frequency, light, and electromagnetic force.

Propagating waveforms are defined by a relationship that associates Wavelength with Frequency and Velocity,

$$v = f\lambda \tag{2}$$

We will call this the *Wavelength equation*. In this equation, Velocity, Frequency, and Wavelength are all Rates. Velocity is defined as the amount of distance per unit of time (e.g., meters per second), Frequency is defined as the number of oscillations per unit of time (e.g., cycles per second), and Wavelength is defined as the amount of distance per unit of oscillation (e.g., meters per cycle). While not obvious, foundational researchers such as Maxwell, Lorentz, and Einstein treat Eqs. (1) and (2) synonymously because of their expression of wavelength as length and frequency as inversed time [12, 13].

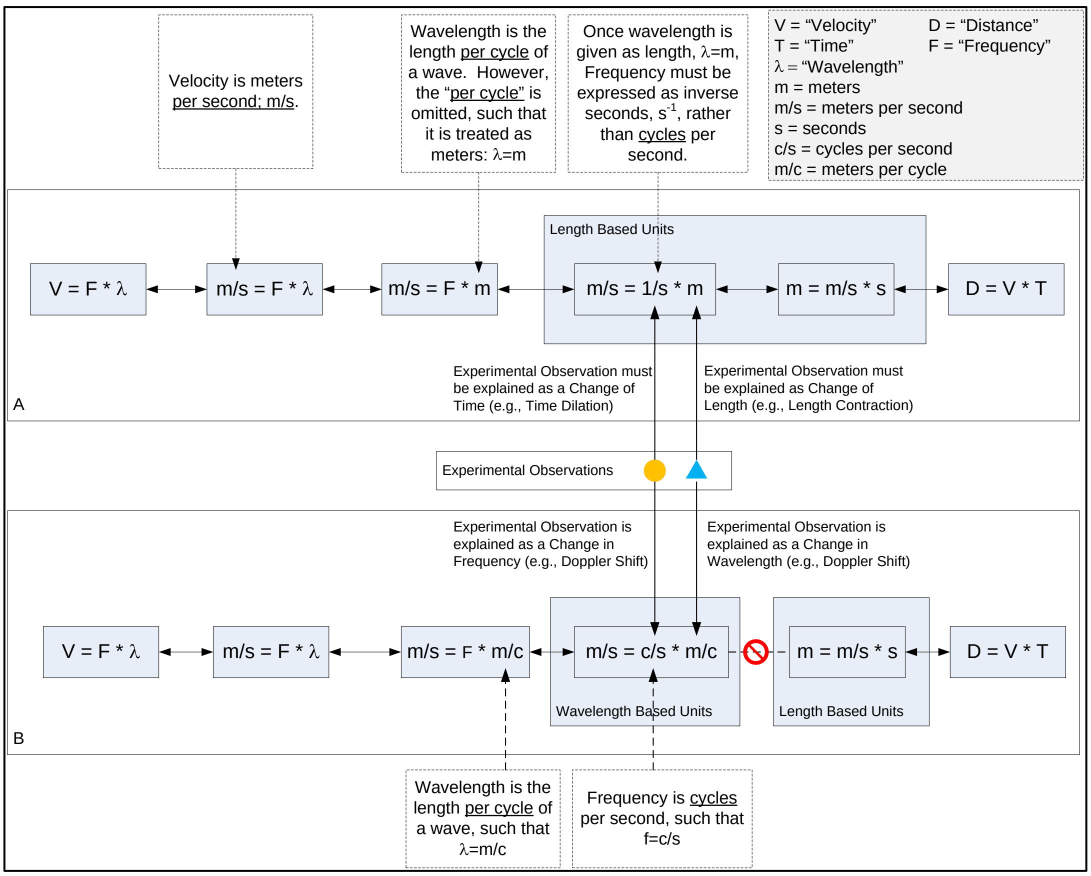

Fig. 1. Relationship of *length equations* and *wavelength equations*. In 1(A), the length and wavelengths equations are treated synonymously – as the length equation. In 1(B), the length and wavelength equations are treated as two separate equations.

As illustrated in Fig. 1(A), Eqs. (1) and (2) are treated as the same equation when λ is interpreted as Length (e.g., meters) rather than as Wavelength (e.g., meters per cycle). Once this occurs, Frequency must be interpreted as inversed seconds, or 1/Time (e.g., seconds^{-1}). When interpreted in this manner, the mathematical expression of Frequency ignores the need to place cycles in the numerator. This results in the mistreatment of the *Wavelength equation* as if it were the *Length equation*.

In Fig. 1(B), both the *Wavelength and Length equations* are denoted with their proper units. Wavelength, Frequency, and Velocity are all Rates in the *Wavelength equation*, differentiating it from the *Length equation*. Thus, the two equations are not mistaken for one another, and both length based and wavelength based equations are appropriately used in the resulting theoretical model.

The *Wavelength and Length equations* differ in two important ways, which cannot be reflected in any theoretical model that does not differentiate between these two types. First, as illustrated in Fig. 2, the wavelength based equations are averaged rather than added. Thus, *Length and Wavelength equations* will yield different mathematical expected results, or different actual results when the equations are used to interpret experimental raw data. Second, as highlighted in Fig. 1, where the circle and triangle shapes in the center of the diagram represent experimental observations, the theoretical explanations of the observations will be different. In a length based model, the circle observation is explained as a *change in time*, whereas in a wavelength based model, it is explained as a *change in frequency*. Similarly, the triangle observation is explained as a *change in length* in a length based model and as a *change in wavelength* in a wavelength based model. Relativity theory explains the changes in length and time as *length contraction* and *time dilation*, respectively. Wavelength based models, such as the author's Modern Classical Mechanics model, explain the change in frequency and wavelength as Doppler Effects. Thus, the distinction between wavelength and length is an important distinction between the two theoretical classes because it explains why theories like Relativity require time dilation and length contraction, and why models such as Modern Classical Mechanics do not. We must now examine why the two approaches yield different predictive equations.

The bottom of Fig. 2 illustrates the mathematical derivation of the equation used in the Michelson-Morley experiment. It begins with the equations for the approaching and receding Doppler shifts. In a length based model, these equations are **added** to one another and used to compute the experiment's expected result. Michelson and Morley found that their length based interpretation of their raw data did not produce the result of 30 km/s, but instead produced 5 to 8 km/s. Lorentz offered a length based **adjustment** to correct the Michelson and Morley equation, offering a better fit between the expected and actual results. While Einstein's equations differ slightly from Lorentz's, he performed the same mathematical adjustment in creating his equation. As a result, the Relativity equations are quantitatively better than those used by Michelson and Morley, as objectively determined by measuring the amount of error between the theoretical expected results and experimental actual results.

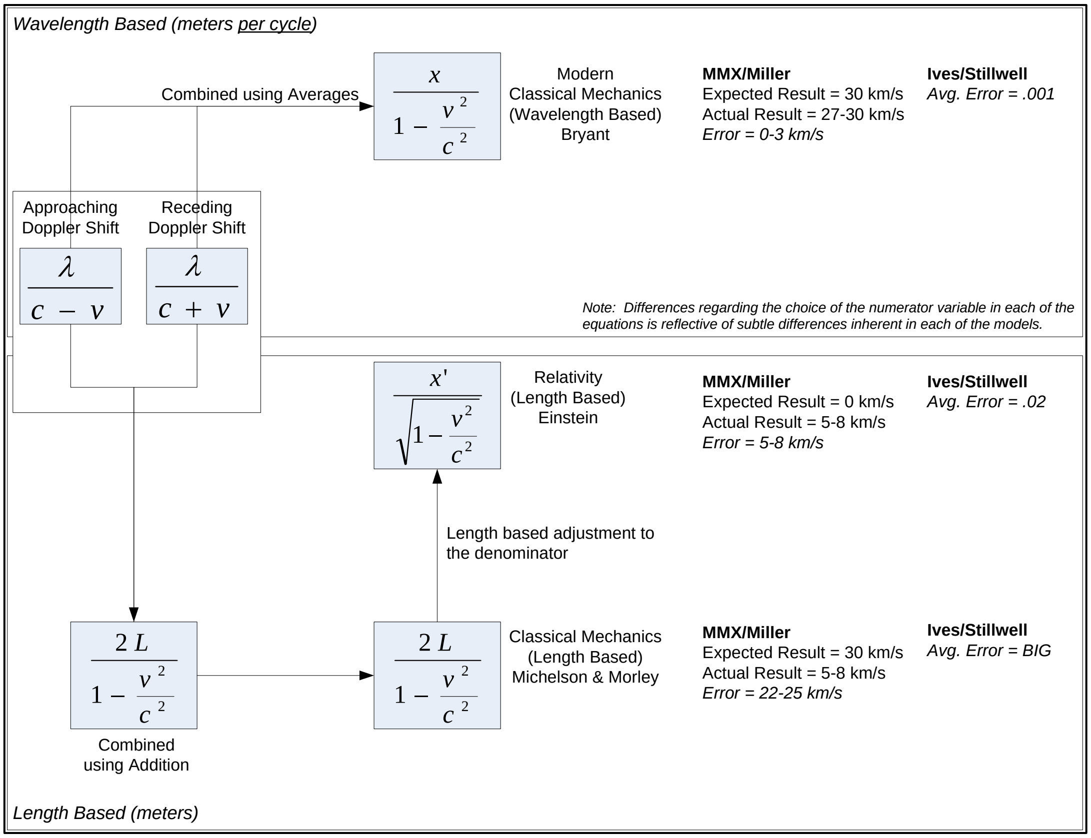

Figure 2. Wavelength and length based equations, expected results, actual results, and model accuracy (as determined by the amount of error). The wavelength based equations outperform the length based equations.

The equations associated with wavelength based models (e.g., Modern Classical Mechanics), as illustrated in the top of Fig. 2, **average** the approaching and receding Doppler equations. This is not an adjustment to the length based equations used by Michelson and Morley, nor is this an adjustment to the equations used by Lorentz or Einstein. Rather, the equation is derived from the Superposition of Waves principle, which averages wavelengths and frequencies, rather than adding them. The result is that the wavelength based equations produce quantitatively better results than those produced by the length based equations.

3. Understanding the Twin Paradox

There are two important points that must be highlighted before explaining the source of the Twin Paradox. First, the *wavelength equation* can be generalized so that it applies to all waves, not just light and electromagnetic force. This is accomplished by using the variable w in place of the constant c in the equation, where w is the propagation speed of the medium under consideration. Second, because the equation applies to all waves and wave mediums, we can use an example that can be readily grasp and understand by a larger audience. With this context set, we can now explain the source of the Twin Paradox: The mistreatment of Wavelength and Frequency as if they were Length and Time. As shown in Fig. 2., because the moving system equations are the average of the approaching and receding Doppler shift equations, we simply use one of these shifts – the approaching shift, due to its dominance in the average – to illustrate the paradox.

Consider two people; Person A is at a train station and Person B is on the train. They both have a horn and a frequency counter. Together, they decide the best way to synchronize and keep time is to count the number of beats, or oscillations, of the other person's horn sound. They agree to a standard – that every 1000 beats (or oscillations) represents one second. In Case 1, the train is stationary. Person A blows his horn, which is picked up on both frequency counters. After 1000 beats, both clocks increment by one second, at the same time. (For the purpose of this example, we can safely ignore propagation delays.) The same scenario occurs when Person B blows his horn – both clocks increment at the same time. Based on this experiment, both parties agree that they have a good timekeeping system.

Now consider Case 2, where the train is moving away from the station at velocity v. Person A, at the train station, blows his horn, which is picked up on both frequency counters. However, due to the Doppler shift and resulting lower frequency observed on the train, the frequency counter on the *train* is incrementing more slowly than the frequency counter at the station. In other words, the frequency counter on the train will take longer to reach 1000 beats than its counterpart at the station: The clock on the Train is *running slower*. While a wavelength based model explains this behavior as reception distortion due to Doppler shift, Einstein's length based model explains this effect as time dilation.

As a continuation of Case 2, consider the situation where Person B, who is on the train, blows his horn. The sound is picked up on both frequency counters. In this case, again due to the Doppler shift, the frequency counter at the *station* will run slower than its counterpart on the train. Once again, a wavelength based model explains this using Doppler shifts, while Einstein's length based theory explains it as time dilation. However, the length based explanation also creates a new artifact, or *paradox: From each person's perspective, his clock is correct and it is the other person's clock that is running slowly.* While this paradox is manifest in a length based model, it does not exist in a wavelength based model where it is explained as a manifestation of the Doppler Effect, or a change in frequency, rather than as a change in time. We have now shown that when wavelength based observations are interpreted using a length based model, that model must explain a change in time (e.g., time dilation), a change in length (e.g., length contraction), and produces accompanying paradoxes (e.g., the Twin Paradox) as a side effect.

4. Conclusion

Moving Systems Equations, as developed and used by Michelson and Morley, Maxwell, Lorentz, and Einstein, are all based on length based equations. Due to the mistreatment of wavelength and frequency as length and time, respectively, the resulting equations and models are inherently length based.

Experimental phenomena can be observed as falling into two categories: length based and wavelength based. When wavelength based experiments, such as Michelson-Morley or Ives-Stillwell are evaluated using wavelength based models, they yield quantitatively better results than when evaluated using equations associated with length based models. Additionally, wavelength based models are easier to understand because they do not require non intuitive concepts such as time dilation or length contraction, nor do they produce side effects like the Twin Paradox.

The Twin Paradox, or paradoxes with similar characteristics, will be inherent in any length based model that tries to explain wavelength observations. Such interpretations will also require explanations for the theoretical *change in length* and the *change in time*; which Einstein does using length contraction and time dilation. However, when wavelength based observations are explained using a wavelength based model, such explanations are not necessary. As a result, a wavelength based model does not enable the introduction of a paradox like the Twin Paradox. Wavelength based models provide a foundation for alternative theories that distinguishes between wavelength and length, where the appropriate use of both equations yield the best mathematical results.

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